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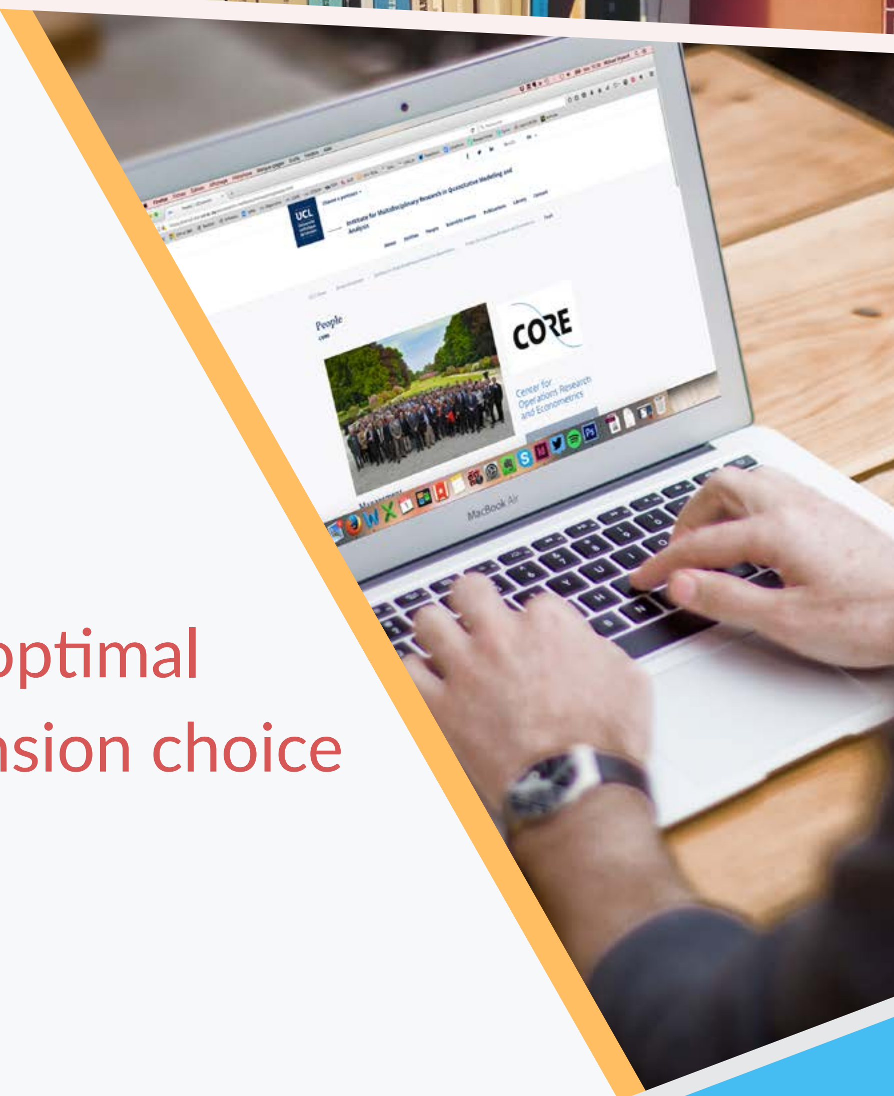

An intermediary's optimal geographical expansion choice under uncertainty

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An intermediary's optimal geographical expansion choice under uncertainty*

Mélanie Lefèvre† and Joe Tharakan‡

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Abstract

High transport costs to reach markets and obtaining low prices on these markets make it difficult for small-scale farmers in developing countries to market their production. Geographically isolated farmers therefore often have to rely on intermediaries to transport and sell their output on markets. To collect output from farmers, these intermediaries have to make investments while facing uncertainty due to the volatility of agricultural prices on world markets. Using real options, we establish the optimal investment strategy for an intermediary in terms of when to invest and with how many geographically dispersed farmers to contract with. We also establish how, after the initial investment, the intermediary should optimally further expand its collection area. We determine what public authorities could do to encourage the emergence of intermediaries who collect production from isolated farmers. Finally, we apply our framework to analyze investment decisions made by intermediaries in the milk sector in Senegal.

Keywords: Intermediary, transport costs, real options
JEL codes: O18, Q13, R42

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1 Introduction

In developing countries, agricultural production comes mainly from smallholder farming or family farming. These farms constitute about 80 percent of African agriculture. 500 million of them provide income to about two-thirds of the 3 billion rural people in the world (FAO, 2008). However, these farms are generally geographically dispersed in rural areas that are far from the areas of consumption. Inadequate transport infrastructure and significant distances between areas of production and areas of consumption hamper farmers' access to the market. High transport costs have been shown to lead to lower sales (HOLLOWAY et al., 2000) and input use (STAAL et al., 2002). STIFEL and MINTEN (2008) found a strong negative relationship between productivity and isolation. They show that the reduction of transport costs increases the use of various inputs such as fertilizer, as well as rice production per acre. Similar effects have been found in Bangladesh (AHMED and HOSSAIN, 1990) and India (BINSWANGER et al., 1993). Evidence also suggests that high transport costs are associated with lower income. In Nepal, JACOBY (2000) found a negative effect of transport costs on the farm profit: a 10% longer travel time causes the maximal profit that can be earned on a hectare of land to be reduced by 2.2%. Although improvements in infrastucture have been made in recent years, the poorest farmers' transport cost has not always decreased. In Nepal, for instance, 19.4% of those in the poorest quintile had access to paved road within 30 minutes in 2003-04 (CBS, 2004), while this proportion was 29.67% in 1995-96 (CBS, 1996). Moreover, fuel prices continue to increase. In Senegal for example, pump price for diesel fuel was equivalent to 0.48 USD in 1998 (GTZ, 1999) and to 1.34 USD in 2010 (GTZ, 2010).

In this context, the presence of intermediaries, such as private traders, retailers, agribusinesses, cooperatives or food processing companies can improve farmers' access to the market. Intermediaries have some advantage over the farmers to sell the products to the markets. Indeed, often a technology exists that allows the goods to be transported at a lower cost (for instance, the use of more efficient means of transport, such as trucks, or processing that reduces volume and/or perishability of the product, etc.). However, this entails an important fixed cost, which cannot be borne by each farmer alone. Evidence suggests that agriculture in developing countries is increasingly characterized by small-holder farmers producing commodities on contract with agro-industrial firms (IFAD, 2003). In Mozambique, 12% of the rural population is working in contract with local enterprises often affiliated with international companies. In Kenya, 85% of sugar cane production depends on small-scale farmers who provide their production to sugar companies.

As intermediaries potentially improve isolated farmers' market access, and hence reduce their poverty, we analyze under what conditions these intermediaries will find it profitable to enter the market and whether policy recommendations can be made to encourage their entry. On the one hand, they need to make costly irreversible investments in order to deal with farmers who are geographically isolated and dispersed. On the other hand, prices of food products are characterized by important volatility which leads to uncertainty and creates an environment, which tends to discourage investment by profit-seeking agents. This price volatility (measured by standard deviations of logarithmic changes in monthly average real prices) in the period 1990-2009 is estimated to be 14% for beef, 25% for sugar, 34% for coconut oil, 45% for oranges and even up to 65% for bananas (GILBERT and MORGAN, 2010).

In this paper, we establish when it is beneficial to invest to become an intermediary who collects the output from farmers dispersed over a certain geographical area, transforms and sells it on an urban market on which the output price fluctuate. This means that we study the profitability of an intermediary's investment opportunity in the geographical scale of its activity in a context of market price uncertainty The contributions in the literature on this issue are limited. LOFGREN (1992) establishes the optimal size of a collection area for a spatial monopsonist in the context of uncertainty. However, his analysis is static. Analysing this question in a dynamic framework is important for two reasons. First, as the output price can change significantly over time, the timing of the initial entry is crucial. If an intermediary enters when the price is too low, it may not recover its initial investment. Second, we show that the initial optimal size depends on whether or not it is possible to expand the collection area in the future. Unlike some investments (e.g. investment in a nuclear power plant), very often a collection area can be expanded in the future. Given the irreversible nature of the investment and the uncertainty linked to the agricultural prices volatility, we use real option theory (DIXIT and PINDYCK, 1994) to establish the optimal investment strategy for such an intermediary. Real options have been applied to different issues. Regarding the optimal strategy in terms of size of a collection area, parallels can be drawn with the literature on capacity choice. Several models of capacity choice have been developed in the real options literature. There are incremental investment models, such as PINDYCK (1988) and DIXIT (1995), as well as models of fixed capacity choice such as DANGL (1999) or BØCKMAN et al. (2008).

In what follows we determine the optimal investment strategy for an intermediary who buys an input from geographically dispersed farmers and who sells this transformed input on a market characterized by price volatility. We determine at what price it is optimal for the intermediary to invest and we also establish what is the optimal initial size of collection area. The higher this optimal price, the less likely the entry by the intermediary on the market; the bigger the collection area, the more the farmers that will benefit from the presence of this intermediary. We also establish what the optimal expansion policy is for the intermediary. Although some farmers might not be included in the collection area initially, they might be in the future if the environment turns out to be more favourable. The optimal expansion policy allows us to establish how likely these farmers will be included in the future. Our analysis also establishes what factors influence the optimal entry strategy and optimal expansion strategy.

If their objective is to improve farmers' access to the market, public authorities or donors, rather than getting directly involved in the collection activity, may want to help private intermediaries invest earlier and in a larger collection area. We compare the impact of various policy measures that can be implemented. Measures that induce a larger number of farmers being included in the project benefit remote farmers. Measures that lead to a shorter investment delay benefit farmers that are included in the area. The impact of measures, such as help for initial investment or aid for investment in collection area expansion, on the optimal investment strategy of the intermediary is not obvious a priori as size and timing are affected in opposite ways. Depending on the donor's objective, we discuss the efficiency of these different measures.

Using information on the milk sector in Senegal, we simulate these effects in a real context. In Senegal, in rural areas, 90% of the households own cattle. Hence, local dairy sector expansion would allow to increase and secure income for a large part of the population. However, farmers are geographically dispersed over large rural areas while consumers are mainly concentrated in Dakar, hundreds of kilometers away from some areas of production. The poor road infrastructure and the high perishability of milk lead to high transports costs. Moreover, the market prices are mainly driven by low and volatile international prices. In this context, farmers are not able to sell the milk by themselves on the final market. Since the nineties, we see the emergence of small-scale firms, called "mini-laiteries," that buy milk from the farmers, transform or pack it, and sell it to the final market.

We apply our framework to two of them.

The paper is structured as follows. The next section develops the theoretical model regarding investment timing and size choice, taking into account the possibility of including at a later date other farmers after the initial investment has been made. Section 3 develops a specific example and discusses some additional comparative statics results. Section 4 is devoted to study cases in the milk sector in Senegal. Section 5 concludes.

2 The model

We consider a situation in which farmers are geographically dispersed and produce an agricultural product. This agricultural product is sold on a market which can be, for instance, an urban center. The price on the urban market for the good is assumed to be stochastic and influenced by the price on world markets. We assume that the market price evolves following a geometric Brownian motion:

$$dp_t = \alpha \; p_t \; dt + \; \sigma \; p_t \; dz \tag{2.1}$$

where α is the drift and σ is the volatility of price p, $dz = \varepsilon\sqrt{dt}$ (where $\varepsilon \sim N(0,1)$ is a white noise) is a Wiener increment, which satisfies $E[dz] = 0$ and $Var[dz] = dt$. The deterministic part $\alpha p_t dt$ represents the trend of world prices and the stochastic part $\sigma p_t dz$ represents the volatility of the market price. The assumption of the geometric Brownian motion seems reasonable for agricultural products including milk. TURVEY and POWER (2006) have performed a test for an ordinary Brownian motion on historical data from 17 commodity futures contracts and have found that the null hypothesis of ordinary Brownian motion cannot be rejected for 14 of the 17 series. Fluid Milk is shown to be consistent with a geometric Brownian motion at all confidence levels.

A risk-neutral intermediary collects the product from the geographically dispersed farmers, pays each farmer a price, transforms the product and sells it on the market. The intermediary pays the farmers a price which is a function of the market price p_t. By assumption, the farmers' supply is price inelastic. This means that if an intermediary wants to increase the quantity it collects, it has to go out further which increases its collection cost. We assume that collecting the good further away is more difficult and leads to an increasing marginal cost of collecting. The transformation of the product leads to a variable cost. Time is continuous, $t \in [0,\infty)$ and time index t is suppressed if not necessary. At each time t, with a market price p_t and a collection area of size R_t the intermediary's operating profit can be written as $\pi(p_t, R_t)$. We have $\pi_p > 0, \pi_R > 0$ and $\pi_{Rp} > 0$. Reflecting the fact that it is more costly to collect the agricultural good from further away we assume $\pi_{RR} < 0$. In addition, we have $\pi_{pp} \geq 0$ and $\pi_{Rpp} \geq 0$. To reflect the fact that the farmers' supply is price inelastic, we assume that if strictly positive these derivatives are not too large. Furthermore, we assume that while they are geographically dispersed, farmers are not too different in terms of their characteristics and the quantity they supply.

To set up a collection area, investments have to be made and these costs are assumed to be sunk and hence irreversible. The investment cost in a collection area has two components. First, there is a fixed cost I which does not depend on the size of the collection area (this can be the cost of building the plant, etc.). Second, there is a cost which increases with the size of the collection area. Indeed, including more farmers in the collection area is costly. This can be due to search and information costs necessary for concluding the contract with farmers. An illustration from the example of the milk sector in Senegal is the following. One way to attract farmers to the network is to encourage artificial inseminations of cows in order to obtain cows with a higher milk production. However in that case, sheds also have to be constructed to protect these animals that cannot resist high temperatures. Training sessions about hygiene, animal welfare, and animal health have to be organized to guarantee the quality of the product. We assume that space is homogeneous and that farmers are uniformly distributed. Hence this expansion cost κ is constant per unit of distance. Thus the initial investment cost for a collection area of size R is $I + \kappa R$. Furthermore, the price might evolve favourably after an initial investment is made and hence the intermediary might want to extend the size of the collection area. In that case, each further expansion of the area costs also κ per additional unit of distance.

We establish the optimal investment strategy for an intermediary who faces a stochastic market price. Both for the initial investment as well as for the expansion investments, the optimal strategy is of the threshold type: the investment is carried out if the price reaches a price threshold. Hence, the optimal investment strategy is described by three elements: (i) a threshold price p^* above which it is optimal for the intermediary to initially invest in a collection area, (ii) the initial size R^* of the collection area that the intermediary chooses when the stochastic market price crosses the threshold p^*, and (iii) the price threshold curve $\hat{p}(R)$ which indicates for each size R what critical price has to be reached in order for it to be optimal to increase the collection area size to R after the intermediary's entry.

We use real options to determine the optimal investment strategy. The real options approach is based on the fact that an opportunity to invest can be viewed as a financial call option: the possibility (or "option"), but not the obligation, to undertake an investment. Given that the intermediary can decide when to invest and that there is no competition for this opportunity, it can be viewed more specifically as a perpetual American call option. If investment is irreversible and future revenues are uncertain, this possibility to wait has a certain value because once the investment is carried out (the option is exercised), the firm cannot disinvest immediately if the market conditions deteriorate. When postponing the investment (that is, keeping the option to invest) the firm not only obtains the "capital appreciation" of the non-invested money, but also avoids future losses. Real option theory tells us that the project should be undertaken when the stochastic price reaches a particular upper threshold, otherwise the firm's best strategy is to wait. To determine this threshold, we need an expression for the option to invest F and an expression for the value V of the project once the investment has been carried out. The threshold has to satisfy a value-matching condition and a smooth-pasting condition (see DIXIT and PINDYCK, 1994). As already mentioned, after this initial investment, the firm can decide to expand the collection area. Hence, the firm not only faces a trade-off for the initial investment, but also for each investment in the geographical expansion of the collection area. In our model we take into account the value of the option to increase the collection area in future periods. The initial investment leads to a value V of the intermediary which depends on the size of the collection area as well as on the option to increase this geographical area in the future.

We proceed as follows. As the price threshold for the initial investment depends on both the value of the intermediary and the option value, we start by determining an expression for V and an expression for F. As the value V of the intermediary is a function of not only future expected profits on the current size of the collection area but also on the future expansion of that collection area, we establish the optimal expansion strategy for the intermediary. We show that the optimal expansion strategy takes the form a price threshold $\hat{p}(R)$ that has to be reached in order to expand the collection area a size R. Finally, we establish the optimal initial size of the collection area and the overall optimal investment strategy of the firm.

2.1 Value of an installed intermediary

Denote by $V(p,R)$ the value of an installed intermediary with a collection area of size R when the current price is p. This is the discounted expected profit the intermediary obtains if it follows the optimal expansion policy:

$$V(p,R) = \max_{\{R_t\}} E_0 \int_0^{\infty} \left[e^{rt}\left(\pi(p_t,R_t)dt - \kappa dR_t\right)\right]$$

where κ is the per-unit cost of the expansion of the collection area, R_t is a nondecreasing function and E_0 represents the expectation conditional on the information available at the current time. Consider a short period of time dt such that the probability of reaching a price at which expansion would be profitable is negligible. This value $V(p,R)$ can be written as the sum of the current operating profit over a time interval of length dt and the continuation value (the expected discounted value of future operating profits) after a length of time dt:

$$V(p,R) \approx \pi(p,R)dt + E_0\left(V(p+dp,R)e^{-\rho dt}\right)$$

where $e^{-\rho dt}$ is the discount factor.

Using a Taylor-MacLaurin expansion, applying Ito's lemma and taking the limit $dt \to 0$, this yields the following non-homogeneous differential equation:[1]

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 V(p,R)}{\partial p^2} + \alpha p \frac{\partial V(p,R)}{\partial p} - \rho V(p,R) + \pi(p,R) = 0 \tag{2.2}$$

Appendix C shows that the particular solution to this non-homogeneous differential equation is given by:

$$V(p,R) = B_1(R)p^{\beta_1} + B_2(R)p^{\beta_2} + g(p,R) \tag{2.3}$$

where $B_1(R)$ and $B_2(R)$ are "constants" to be determined, while $\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}$ and $\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}$ are, respectively, the positive and negative root of the characteristic equation. One of the boundary conditions is $V(0,R) = 0$: for all sizes R, if price p goes to zero, then the intermediary's value goes to zero as zero is an absorption state for p. Therefore, the coefficient B_2, corresponding to the negative root β_2, should be equal to zero, such that (2.3) becomes:

$$V(p,R) = B_1(R)p^{\beta_1} + g(p,R) \tag{2.4}$$

The interpretation of (2.4) is the following. The last term represents the expected present value of the profit the intermediary would obtain if it kept the size of the collection area constant at the level R forever. $B_1(R)p^{\beta_1}$ is the value of the intermediary's options to expand this area in the future. Constant $B_1(R)$ is determined by considering the other boundary $\hat{p}(R)$ above which it is optimal to expand. Both $B_1(R)p^{\beta_1}$ and $\hat{p}(R)$ will be established simultaneously as part of the solution.

[1] See Appendix A for a derivation.

2.2 Value of the option to invest F

When a firm has the opportunity to invest in a collection area which will yield a value $V(p,R)$, it holds an option to invest which has a value denoted by $F(p)$. The firm optimally chooses the timing T of the investment and the initial size R of the collection area. Hence, we have

$$F(p) = \max_{T,R} E_0 \left[e^{\rho T} (V(p_T, R) - I - \kappa R) \right]$$

As long as the investment is not undertaken, holding the option to invest yields no cash flow and thus the only return it yields is its "capital appreciation." Consider a short period of time dt such that the probability of reaching a price at which it is profitable to exercise the option is negligible. We can then write:

$$F(p) \approx e^{-\rho dt} E_0[F(p+dp)] \tag{2.5}$$

Using a Taylor-MacLaurin expansion, applying Ito's lemma and taking the limit $dt \to 0$, $F(p)$ satisfies the following differential equation[2]

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 F(p)}{\partial p^2} + \alpha p \frac{\partial F(p)}{\partial p} - \rho F(p) = 0 \tag{2.6}$$

The general solution to this equation can be written as $F(p) = A_1 p^{\beta_1} + A_2 p^{\beta_2}$. This solution is valid over the range of prices for which it is optimal to hold the option. This range is defined by boundary conditions. One natural boundary condition is 0. Since $p = 0$ is an absorbing barrier, the option to invest has no value for very small values of p. This indicates that the constant A_2, corresponding to the negative root β_2, must be equal to zero:

$$F(p) = A_1 p^{\beta_1} \tag{2.7}$$

where A_1 is a constant to be determined.

The other boundary of that region is p^*, the price at which it is optimal to exercise the option (i.e. to invest). This boundary is a "free boundary": p^* is endogenous and must be determined simultaneously with $F(p)$. This means that A_1 will be determined as a part of the solution, simultaneously with threshold p^* above which it is optimal to invest.

2.3 The intermediary's optimal expansion policy

Since function $\pi(p,R)$ is increasing and concave in R for each p and is increasing in p and since the expansion cost is proportional to the size of the expansion, the optimal policy will be of the threshold type (Stokey, 2009). This means that when the price p reaches a certain threshold $\hat{p}(R)$, investment in expansion is carried out immediately such that the collection area reaches size R. We establish the price threshold $\hat{p}(R)$ that price p must reach in order for the optimal collection size to be R. This price threshold has to satisfy two boundary conditions: a value-matching condition and a smooth-pasting condition (Dixit and Pindyck, 1994).

[2] For a proof see Appendix B

There are different ways in which this can be shown. Denote by $f(p,R)$ the value of the option to expand for an intermediary with a collection area of size R. Following a similar reasoning as developed in Section 2.2 it is easy to show that this can be written as $f(p,R) = b_1(R)p^{\beta_1}$. The additional value obtained by expanding is given by $\partial g(p,R)/\partial R \quad (\equiv v(p,R))$. Price threshold $\hat{p}(R)$ has to satisfy a value-matching condition and a smooth-pasting condition. The value-matching condition is given by $f(\hat{p}(R),R) = v(\hat{p}(R),R) - \kappa$ while the smooth-pasting condition is given by $\partial f(\hat{p}(R),R)/\partial p = \partial v(\hat{p}(R),R)/\partial p$. Another way to arrive at these conditions is to start from the value function given by (2.4). In that case the value-matching condition is given by $\partial V(\hat{p}(R),R)/\partial R = \kappa$ while the smooth-pasting condition is given by $\partial^2 V(\hat{p}(R),R)/\partial R \partial p = 0$.

The value-matching condition can be written as

$$\frac{\partial g(\hat{p}(R),R)}{\partial R} = \kappa + b_1(R)\hat{p}(R)^{\beta_1} \tag{2.8}$$

with $b_1(R) = -\partial B_1(R)/\partial R$. Equation (2.8) says that the firm should expand the size until the value of marginal unit of size is equal to the cost of this marginal unit. However, this cost not only includes the purchase cost κ but also includes the opportunity cost $f(p,R)$ of sacrificing the option to invest in the marginal unit. The smooth-pasting condition can be written as:

$$\frac{\partial^2 g(\hat{p}(R),R)}{\partial R \partial p} = \beta_1 b_1(R)\hat{p}(R)^{\beta_1 - 1} \tag{2.9}$$

Combining these conditions yields an expression which implicitly defines the price threshold $\hat{p}(R)$ that must be reached to expand the size up to R:

$$\frac{\partial g(\hat{p}(R),R)}{\partial R} = \kappa + \frac{\hat{p}(R)}{\beta_1}\frac{\partial^2 g(\hat{p}(R),R)}{\partial R \partial p} \tag{2.10}$$

As the last term is positive, this condition shows us that the optimal expansion policy implies the intermediary should not expand its collection area until the marginal benefit equals the marginal cost of expansion κ. The size of the collection area should be smaller than that implied by the equality of the marginal benefit and marginal cost. The difference is due to the last term which, through β_1, depends on σ. The optimal investment policy of an investor is to take into account the variability of prices, even if he is risk-neutral. The reason is that there is the risk that the price might decrease in the future. Hence, an investor will optimally wait unit the marginal benefit is sufficiently higher than the marginal cost before investing expanding the collection area.

We show that uncertainty delays capacity expansion, i.e. $d\hat{p}/d\sigma > 0$. Likewise, an increase in the marginal cost of expansion κ leads to a delay in the expansion of the collection area.

Proposition 1. *The optimal expansion strategy is to invest whenever $p > \hat{p}(R)$ when the collection size is R. The optimal size is larger when price p reaches higher levels. Higher uncertainty and higher expansion cost reduce the optimal collection area size.*

Proof. See Appendix D. □

2.4 Entry price threshold $p^*(R)$

We establish the threshold price $p^*(R)$ above which it is optimal to invest in a collection area of size R. The investment strategy is determined by the following trade-off. On one hand, investing later saves the interest on the investment cost $I + \kappa R$. On the other hand, investing now yields an immediate cash flow plus the opportunity to expand the size (so to increase the future cash flow) but eliminates the opportunity to avoid losses if the market price decreases. This investment strategy satisfies a value-matching condition and a smooth-pasting condition.

The value-matching condition indicates that at the threshold price $p^*(R)$ the firm is indifferent between investing in a project of size R, and not investing, that is: $F(p^*(R)) = V(p^*(R), R) - \kappa R - I$. The smooth-pasting condition is given by $\partial F(p^*(R))/\partial p = \partial V(p^*(R), R)/\partial p$. This yields:

$$g(p^*(R), R) + B_1(R)p^*(R)^{\beta_1} = \kappa R + I + A_1 p^*(R)^{\beta_1} \tag{2.11}$$

$$\frac{\partial g(p^*(R), R)}{\beta_1 \partial p} + B_1(R)p^*(R)^{\beta_1 - 1} = A_1 p^*(R)^{\beta_1 - 1} \tag{2.12}$$

Combining (2.11) and (2.12), we obtain an expression which defines implicitly $p^*(R)$:

$$g(p^*(R), R) = \kappa R + I + \frac{p^*(R)}{\beta_1} \frac{\partial g(p^*(R), R)}{\partial p} \tag{2.13}$$

As the last term is positive this condition shows that the optimal entry strategy is to wait until the benefit of investing in a collection area of size R is (much) larger than the cost of the investment.

Proposition 2. *For a collection area of size R, the optimal entry policy is to wait till the price reaches price $p^*(R)$. This threshold is higher for small collection areas as well as large collection areas. The cost of investment I, the cost of expansion κ and uncertainty all delay entry.*

Proof. See Appendix E. □

The function $p^*(R)$ is U-shaped. This is explained by the fact that for small collection areas the variable cost of collection is low, but the average fixed cost high. The inverse is true for large collection areas.

2.5 Optimal investment strategy

The threshold for investment for any given size R is given by $p^*(R)$ defined by (2.13) while the optimal size is for any given market price p is given by $\hat{R}(p)$ defined by (2.10). The solution to this system of two equations in the two unknowns p and R gives us p^* and R^* which are the entry price threshold and the initial optimal size. Above this entry price threshold p^* it is optimal to invest in a project of size R^*. We can show that the intersection of the two curves exist and is unique. After the initial investment, the intermediary expands R when p increases, following the curve $\hat{R}(p)$ given by (2.10).

With the elements developed earlier we can establish the optimal investment policy of the intermediary.

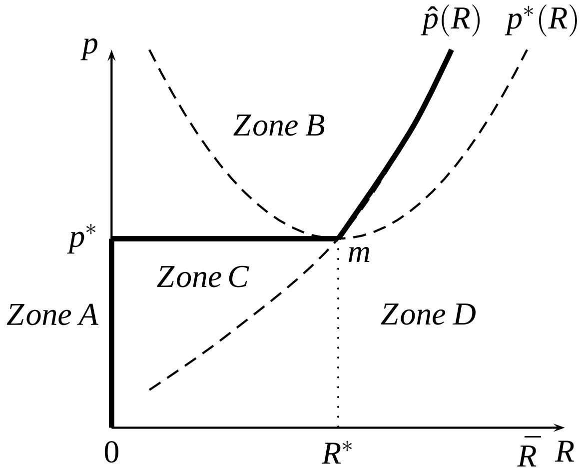


Figure 1: Optimal investment policy

Proposition 3. *An intermediary should enter the first time the market price p crosses threshold p^* and invest in a collection area of size R^*. If the market price $p > p^*$, then the intermediary should invest immediately in a collection area of size $\hat{R}(p)$. As to an intermediary with a collection size R, it should expand its collection area up to $\hat{R}(p)$ whenever market price p crosses threshold $\hat{p}(R)$.*

Proof. See Appendix F. □

The optimal strategy for the intermediary is illustrated in Figure 1. For an intermediary considering entry, the optimal strategy is to wait till the market price reaches p^* and then invest to create a collection area of the size R^*. If the market price p happens to be above p^* when the intermediary is considering its optimal investment policy, then it should immediately invest in a collection area of the size $\hat{R}(p)$. For an intermediary already present with a collection area of size R, three different cases can be distinguished. If the price is above p^* (zone B), then the optimal policy is to expand immediately its collection area up to the size $\hat{R}(p)$. In this case, the optimal size is larger than R^*. If the price happens to be below p^* but above $\hat{p}(R)$, i.e. zone C, then the optimal investment policy is to expand immediately the collection area till $\hat{R}(p)$. Finally, with a collection area of size R, if the price is below $\hat{p}(R)$, i.e. zone D, then the best action is to wait.

3 An example

To illustrate the ideas developed in the previous section we consider a specific case of the model described above.

3.1 Additional assumptions

The intermediary collects the agricultural product from farmers over a collection area of the size R. We assume that farmers are uniformly distributed over space. With a collection area of size R, the intermediary faces a collection cost $T(Q,R)$ where Q represents the total volume collected. Collection cost $T(Q,R)$ increases with the size of the collection area and includes costs such as fuel, driver's wage, etc. It also increases with the volume transported. Hence we have $\partial T(Q,R)/\partial Q > 0$

and $\partial T(Q,R)/\partial R > 0$. In addition, we have $T(0,R) = T(Q,0) = 0$. In each location z, he pays the farmer a price $p_f(z)$ and obtains at that location a quantity $s(p_f(z),z)$. The total quantity Q collected over the whole area is transformed leading to a total cost of $C(Q)$. This cost represents outlays on, for instance, electricity, output packaging, etc. The transformed product is sold on the market at a price p. Hence the profit can be written as:

$$\pi(p,R) = \int_0^R \left((p - p_f(z))s(p_f(z),z)\right) dz - C(Q) - T(Q,R)$$

As collection takes place in a rural area where the road infrastructure is poor we assume that there are no scale and distance economies in transport such that $T(Q,R) = \tau QR$ with $\tau > 0$. Hence, the intermediary's operating profit at each period can be written as

$$\pi(p,R) = sR\left((1-\psi)p - c - \tau R\right) \tag{3.1}$$

where $\psi \equiv \int_0^R \left(p_f(z)s(p_f(z),z)\right) dz/pQ$ is the average farmers' "terms of trade", c the average production cost $C(Q)/Q$ and s the average output per distance.

Further assuming that supply s is inelastic and that ψ does not depend on p, using expression (3.1) it is easy to show that, since p evolves as a Geometric Brownian motion, $g(p,R)$ can be written as the following simple expression

$$g(p,R) = sR\left(\frac{(1-\psi)p}{\rho - \alpha} - \frac{c + \tau R}{\rho}\right) \tag{3.2}$$

3.2 Optimal investment strategy

Using (2.10) and (3.2) the price threshold $\hat{p}(R)$ that must be reached to expand the size up to R can be written:

$$\hat{p}(R) = \frac{\beta_1(\rho - \alpha)}{(\beta_1 - 1)(1-\psi)}\left(\frac{\kappa}{s} + \frac{c + 2\tau R}{\rho}\right) \tag{3.3}$$

The interpretation of $\hat{p}(R)$ is the following. Since both $\beta_1/(\beta_1 - 1) > 1$ and $1/(1-\psi) > 1$, the threshold $\hat{p}(R)$ is a multiple of the flow equivalent of the marginal cost of producing an additional unit of output. This cost has three components: an investment, a transformation and a collection cost. Hence it is optimal to wait until the marginal location is sufficiently "in the money" before investing. The term $\beta_1/(\beta_1 - 1)$ reflects the existence of an option value of waiting. The term $1/(1-\psi)$ depends positively on the bargaining power of the farmers; the more important the bargaining power of farmers is (the higher ψ), the more the intermediary is going to wait before investing.

This expression can be used to determine the optimal size $\hat{R}(p)$ for price p. The optimal size is given by the inverse of the threshold curve (3.3):

$$\hat{R}(p) = \frac{\rho}{2\tau}\left(\frac{(\beta_1 - 1)}{\beta_1}\frac{(1-\psi)p}{\rho - \alpha} - \frac{\kappa}{s} - \frac{c}{\rho}\right) \tag{3.4}$$

Although that paper assumes a static environment, a comparison can be made with a result in Lofgren (1992). Lofgren shows that the optimal size for the collection area of a monopsonist under uncertainty should be the size at which the expected revenue is equal to the expected cost. In (3.4), $(1-\psi)p/(\rho - \alpha)$ corresponds to the expected discounted marginal revenue while $(\kappa/s) + (c/\rho)$

corresponds to the discounted marginal cost which does not include the marginal cost of collecting. Since $(\beta_1 - 1)/\beta_1 < 1$, our results show that, even if the intermediary is risk-neutral, the optimal size is smaller than that implied by Lofgren's condition. The reason is that, because it is possible to expand the collection area in the future after the initial investment, we have to take into account the value of the option to further expand this area when establishing the optimal initial size.

Equations (2.8) and (2.9) also allow us to establish the value of $b_1(R)$. Value matching says that, at the optimal size level, the marginal discounted profit has to be equal to the marginal cost of increasing the size. This cost includes monetary cost κ as well as opportunity cost $-b_1(R)p^{\beta_1}$. When the firm exercises its option to install the Rth unit of size, it gives up the marginal option value $-b_1(R)p^{\beta_1}$. With this definition, $b_1(R)$ is negative. Replacing p by (3.3) in the smooth-pasting condition we obtain:

$$b_1(R) = -\frac{s(\beta_1 - 1)^{\beta_1 - 1}(1-\psi)^{\beta_1}}{\beta_1^{\beta_1}(\rho-\alpha)^{\beta_1}} \left(\frac{\kappa}{s} + \frac{c + 2\tau R}{\rho}\right)^{1-\beta_1} \tag{3.5}$$

This allows us to determine $B_1(R)$. Indeed, as $B_1(R)p_1^{\beta}$ represents the value of the intermediary's growth options, it is given by the integration of the marginal value $-b_1(R)p^{\beta_1}$: $B_1(R)p_1^{\beta} = \left(\int_R^{\bar{R}} \left(-\frac{\partial B_1(z)}{\partial z}\right) dz\right) p_1^{\beta}$ where $\bar{R}$ represents the boundary of the rural area. Hence we have

$$B_1(R) = \frac{1}{(\beta_1 - 2)} \frac{\rho}{2\tau} \frac{s(\beta_1 - 1)^{\beta_1 - 1}(1-\psi)^{\beta_1}}{\beta_1^{\beta_1}(\rho-\alpha)^{\beta_1}} \left[\left(\frac{\kappa}{s} + \frac{c+2\tau R}{\rho}\right)^{2-\beta_1} - \left(\frac{\kappa}{s} + \frac{c+2\tau \bar{R}}{\rho}\right)^{2-\beta_1}\right]$$

Using (2.13) and (3.2) the threshold $p^*(R)$ that needs to be reached before the intermediary spends the initial investment cost $I + \kappa R$ in order to invest in a project of size R is given by:

$$p^*(R) = \frac{\beta_1(\rho-\alpha)}{(\beta_1 - 1)(1-\psi)} \left(\frac{\kappa}{s} + \frac{c+\tau R}{\rho} + \frac{I}{sR}\right) \tag{3.6}$$

Threshold $p^*(R)$ is a multiple of the flow equivalent of the average total cost of investing in a collection area of the size R. As before this multiple depends both on the option value of waiting and on the bargaining power of farmers.

The threshold for investment for any given size R is given by $p^*(R)$ defined by (3.6) while the optimal size is for any given market price p is given by $\hat{R}(p)$ defined by (3.4). The solution to this system of two equations in the two unknowns p and R gives us p^* and R^* which are the entry price threshold and the initial optimal size. Above this entry price threshold p^* it is optimal to invest in a project of size R^*. Combining (3.4) and (3.6), the threshold p^* is given by:

$$p^* = \frac{\beta_1(\rho-\alpha)}{(\beta_1 - 1)(1-\psi)} \left(\frac{\kappa}{s} + \frac{c}{\rho} + 2\frac{\tau}{\rho}\sqrt{\frac{\rho I}{\tau s}}\right) \tag{3.7}$$

At that level, the firm establishes the plant (which costs I) with a capacity $\hat{R}(p^*)$ (that costs $\kappa\hat{R}(p^*)$). Combining (3.4) and (3.7), this initial size is given by:

$$R^* \equiv \hat{R}(p^*) = \sqrt{\frac{\rho I}{\tau s}} \tag{3.8}$$

R^* is the size of the collection area which minimizes the discounted average cost $\frac{c+\tau R}{\rho} + \frac{\kappa}{s} + \frac{I}{sR}$ and hence it represents the minimum efficient scale of the collection area. It is also the size which leads to the lowest entry price given by expression (3.6). Hence the optimal strategy for the intermediary is to invest in a collection area which minimizes the per unit cost and hence allows the intermediary to enter as soon as possible. This means that the focus is on the timing of the investment rather than the size. This result is due to the existence of an expansion option. To see this, assume that the expansion option would not be present. In that case the term $B_1(R)$ in (2.4) is equal to zero and once the initial size has been chosen, it cannot be increased. The optimal size is established as follows:

$$\max_R sR\left(\frac{(1-\psi)\,p}{\rho-\alpha} - \frac{c+\tau R}{\rho}\right) - (I+\kappa R) \tag{3.9}$$

The first-order condition is given by:

$$s\left(\frac{(1-\psi)\,p}{\rho-\alpha} - \frac{c+2\tau R}{\rho}\right) = \kappa \tag{3.10}$$

The optimal size for a given price is in this case

$$R^F(p) = \frac{\rho}{2\tau}\left(\frac{(1-\psi)p}{\rho-\alpha} - \frac{\kappa}{s} - \frac{c}{\rho}\right) \tag{3.11}$$

Given that $(\beta_1-1)/\beta_1 < 1$, it is clear that $R^F(p) > \hat{R}(p)$. The absence of the option to expand does not affect $p^*(R)$, the threshold for initial investment for a given size R. The solution to the system of equations (3.6) and (3.11) is given by:

$$R^F = \frac{\rho}{2\tau(\beta_1-2)}\left(\left(\frac{\kappa}{s}+\frac{c}{\rho}\right) + \sqrt{\left(\frac{\kappa}{s}+\frac{c}{\rho}\right)^2 + 4\beta_1(\beta_1-2)\frac{\tau I}{s\rho}}\right) \tag{3.12}$$

It is easy to show that $R^F > R^*$. When the intermediary has no option to expand the size in the future, he faces a trade-off between investing rapidly in a small collection area and waiting in order to invest in a larger area. When he has the option to increase the size, this trade-off disappears as he can invest rapidly in a small collection area and expand it in the future. In this case, minimizing the price threshold for the initial investment is the optimal strategy.

3.3 The trade-off between size and timing

Each change in the external environment of the intermediary has two different effects on the optimal investment policy. On the one hand, there is a "pure" entry price effect: the change induces the firm to advance or postpone the initial investment, for any given initial size. On the other hand, we have a "pure" size effect: given the current price, the optimal size of the collection area is affected by this change. As these two effects can go in opposite directions, the total effect on p^* and R^* is a priori ambiguous. However, for p^*, since the expansion price curve always goes through the minimum of the entry price curve, a change in a parameter affects price p^* only through its effect on the entry threshold. For R^*, the two effects are present and the total effect is a priori ambiguous. For instance, a higher uncertainty tends to delay the investment for any initial size, but also tends to reduce the optimal size, for any given price. Hence, one cannot a priori determine whether R^* will be larger or smaller with a higher level of uncertainty.

Let us define $p^*(R,x) \equiv p^*(R)$ and $\hat{R}(p,x) \equiv \hat{R}(p)$ to reflect the fact that both variables depend on parameter x. The pure entry price effect of a change in a parameter x is given by the first (partial) derivative of (2.13) with respect to parameter x: $\partial p^*(R,x)/\partial x$, that is the effect of x on the entry price threshold curve, for any given R. The pure size effect is given by the first (partial) derivative of (3.4): $\partial \hat{R}(p,x)/\partial x$, that is the effect of x on the optimal size, for any given p. The effect on the threshold for initial investment is given by the derivative of (3.7): dp^*/dx. Finally, the total effect on R^* is given by the derivative of (3.8): dR^*/dx, that is, $(\partial \hat{R}(p,x)/\partial p)(dp^*/dx) + (\partial \hat{R}(p,x)/\partial x)$.[3] The first term represents the indirect effect of the parameter on R^*, that is, the effect due to a change in the entry price. The second term represents the direct effect of the parameter on R^*. Since $(\partial \hat{R}(p,x)/\partial p) > 0$, the indirect effect has the same sign as the entry price effect. Table 1 reports these effects for different parameters of the model.

Table 1: Entry price and size effects

	Entry price effect $\frac{\partial p^*(R,x)}{\partial x}$	Size effect $\frac{\partial \hat{R}(p,x)}{\partial x}$	Effect on p^* $\frac{dp^*}{dx}$	Total effect on R^* $\frac{dR^*}{dx} = \frac{\partial \hat{R}(p^*,x)}{\partial p}\frac{dp^*}{dx} + \frac{\partial \hat{R}(p^*,x)}{\partial x}$
Uncertainty σ	$+$	$-$	$+$	0
Investment cost I	$+$	0	$+$	$+$
Expansion cost κ	$+$	$-$	$+$	0
Transport cost τ	$+$	$-$	$+$	$-$
Bargaining power ψ	$+$	$-$	$+$	0
Supply s	$-$	$+$	$-$	$-$

The formal expressions which correspond to the different signs are given in Appendix G in Table 4.

When the uncertainty is high (i.e. σ is large), the value of the option to invest, that is the option to wait before investing, is also large and thus the opportunity cost of investing is large. This means that when uncertainty is high the firm postpones investments and waits for output price p to be higher before investing. Our results show that when the uncertainty increases, the intermediary chooses to postpone the initial investment ($dp^*/d\sigma > 0$). For any given size of the collection area, $\hat{p}(R)$ is larger: for a given size R the threshold that price p needs to reach is higher. Equivalently, $\hat{R}(p)$ is lower: for any given market price, the optimal size for the firm is smaller. And in order to expand to an even larger R, the price increase required is even larger, making it less likely that remote farmers would be included in the collection area. Interestingly, in this example, the minimum size of the collection area remains the same and is independent of the level of uncertainty ($d\hat{R}(p^*)/d\sigma = 0$). This is explained by the fact that the indirect and direct effects go in opposite directions, and cancel each other out. The direct effect of an increase in uncertainty is to decrease the optimal size. However, this decrease is exactly compensated by the fact that the intermediary waits until the price reaches a higher level, and hence a larger collection area, to enter.

Facing higher transport costs (i.e. larger τ), for a given price the intermediary invests in a smaller collection area ($\partial \hat{R}(p,\tau)/\partial \tau < 0$) as collecting the product on a given area becomes costlier. The

[3] For derivations, see Appendix G.

effect on the price for initial investment p^* is positive: when the transport costs are large, the intermediary waits longer before making the initial investment to compensate for the higher cost. In addition, the effect on the minimum size $\hat{R}(p^*)$ is negative. Hence higher transport costs reduce farmers' access to market in two ways as the intermediary invests later and in a smaller collection area.

The cost I does not influence the optimal size of the collection area. It does influence the entry price: a higher I requires a higher price in order to enter. Investment cost I does require a higher minimum size R. The reason is that, although it does not influence the expansion cost, it does influence the average (fixed) cost, which means that a larger area needs to be covered to collect a sufficient quantity of the product to cover this fixed cost I.

Cost κ influences both the optimal entry price as well as the minimum size as this cost enters both the marginal and average cost. But, as it was the case with σ, a change in κ leaves the minimum size of the collection area unchanged. The direct effect on the minimum size is exactly compensated by the indirect effect which operates through the price. An increase in κ only increases the entry price threshold and hence delays investment by the intermediary.

Facing farmers with low individual supply (low s), the intermediary has to wait for a higher market price before investing in an extension of the collection area becomes profitable ($\hat{p}(R)$ is larger). Hence, the optimal size of the collection area $\hat{R}(p)$ for any given market price p is smaller. In the same way, the intermediary has to wait longer before making the initial investment: for any given initial size R, $p^*(R)$ is higher. This effect is even more important when considering a small initial collection area. Indeed, it is more difficult to cover the investment cost incurred when the total level of production (sR) is low. Hence, facing a low individual supply, the intermediary invests later (p^* is higher) but in a larger initial size in order to compensate for the low level of production caused by the low individual supply (R^* is larger for a lower s).

Similarly to a low s, a high bargaining power for farmers (a high ψ) makes an investment less attractive for the intermediary as it reduces the revenue that it can obtain. The intermediary will wait for a higher price p before investing. However, unlike s, a change in ψ does not influence the minimum size of the collection area.

3.4 Measures to encourage the emergence of intermediaries

High uncertainty and transport costs negatively affect the intermediary's investment decisions and hence reduce farmers' access to the market. Indeed, in both cases, the size of the collection area, hence the number of farmers included, is lower for any given market price ($\hat{R}(p)$ is smaller). Moreover these two elements negatively affect the timing of the investment: the firm has to wait longer before doing the initial investment (p^* is higher). This would not be such a problem if the initial size of the collection area were larger. In this case, the intermediary would simply wait longer in order to make an investment that benefits more farmers. But we have shown that, under high uncertainty, the initial size of the investment is not increased and that under high transport costs, it is even smaller.

To deal with this outcome, a donor has different possibilities. We analyze two of them here. First, he can provide aid in the form of support to the intermediary for the investment I. This could consist in financing part of the investment cost I. Decreasing I has no effect on the optimal size of the collection area for a given market price ($\hat{R}(p)$) or, equivalently, on the market price $\hat{p}(R)$ that has to be reached in order to increase the size. However, it decreases the entry price threshold p^* for initial investment. Since the reduction in I has no impact on the optimal size, the minimum size of the collection area $\hat{R}(p^*)$ is smaller. Only if the market price crosses the entry price threshold that was relevant before the donor's intervention, then the size of the collection area also reaches the level of the initial size before intervention. From the farmers' point of view, a donor financing a part of the investment cost I helps the intermediary to propose a contract to some of them sooner. For the most distant farmers, however, this intervention has no effect, as the price has to cross the without intervention entry price threshold for them to be included.

Second, the aid for the investment may be dedicated to increase the number of farmers included rather than to the initial investment. In this case, the donor finances a part of κ. This increases the optimal size for any given market price. Equivalently, the price that has to be reached to invest in a given size is lower. Moreover, decreasing κ also decreases the threshold for initial investment p^*. The two effects cancel each other out and keep the minimum size $\hat{R}(p^*)$ unchanged. This means that the initial investment takes place sooner, and this without reducing the number of farmers that are included, contrarily to what happens when the donor finances directly the cost I.

Which instrument should be used depends on the donor's objective. For instance, if the donor wants to improve remote farmers' access to the market, the donor should help the intermediary to reduce κ. If his objective is to give more rapidly access to the market for farmers, then either instrument can be used. However, with I there is a trade-off: a lower investment cost leads to a faster entry of the intermediary but with less farmers. With κ the advantage is that the entry is faster but with no decrease in the number of farmers. In terms of farmers' participation, the ideal is to have the intermediary entering faster and with a larger collection area. Neither a reduction in I nor in κ can achieve this. The only parameter change that can achieve both objectives is a decrease in τ: it decreases the entry price threshold while increasing the optimal size of the collection area. Hence an improvement in the rural transport infrastructure is an important measure that public authorities should focus on if the objective is to help the most isolated farmers in rural areas.

4 Case studies: Senegalese milk sector

We apply our theoretical model to the Senegalese milk sector. Although milk consumption is still low compared to the rest of the world, dairy products are now part of the consumption habits of most African households. Currently, the Senegalese demand for dairy products is mainly satisfied by imports, mostly from Europe. This may be explained by two factors. First, most of the Senegalese milk sector is characterized by a pastoral or agro-pastoral system of production. Farmers are distributed in large rural areas, while consumers are concentrated in the main urban centers. The high transport costs prevent the farmers to access the market by themselves. Second, as any agricultural product, milk faces high price volatility. The uncertainty it creates tends to reduce the investments in the sector.

Since the nineties, we have seen the emergence of small-scale processing units that play the role of an intermediary between the farmers and the market. These intermediaries also face high transport costs and uncertainty, but, unlike farmers, are able to support large investment costs. In addition, there are donors who want to improve farmers' access to the market by providing aid to set up such firms.

We analyze two existing processing units. For both processing units, we assess whether the investment they made was right according to our model and determine the possible future growth of the collection area, that is, the improvement of remote farmers' access to the market. As little information is available on these processing units we cannot test whether they have followed the optimal strategy as described above; for example, we do not have information on the different actions taken in the past by these intermediaries. What we can do is to check whether the situation as it is observed is compatible with the optimal investment strategy. For each case we establish the optimal entry price p^* and the optimal initial size for the collection area R^* as well as the optimal expansion strategy in function of price, $\hat{R}(p)$.

We estimate the drift of the milk price distribution process as the mean of the monthly milk price index (IHPC lait) growth rate in Senegal (from November 2005 to May 2010), divided by 30: $\alpha = 0.0001$. As an estimation of σ, we use the square root of the variance of this index, that we divide by 30: $\sigma = 0.01$. Finally, we choose an annual interest rate of 5.5% which corresponds to the rate on Treasury bills of Senegal's Central Bank, which we divide by 365.

For the other parameters, we collect information regarding their values from various sources. The main difficulty we face in establishing the values of the parameters is that, although we know the total investment cost, we do not know the distribution of this total cost between the fixed cost and the variable cost. Hence we do not have information regarding the values of I and κ. Therefore, we proceed as follows. First, we establish what the implied values of the parameters κ and I would be if the observed entry size were the optimal size. We then use these parameter values to establish whether the optimal entry price corresponds to the observed product price at the time of entry. The second exercise we do is see whether there is a distribution of the total cost between the fixed cost and the variable cost such that the observed entry price and entry size could be optimal.

4.1 Le Fermier

The small-scale milk processing unit "Le Fermier" is located at Kolda in Southern Senegal. This unit produces sour milk and pasteurized milk, using fresh milk from the farmers located in the countryside around Kolda. The products are mainly sold to the consumers in Kolda. Le Fermier is the most important processing unit in the region of Kolda. It treats more than 40% of the milk collected in this region (DIEYE, 2006). Since 2001, Le Fermier is involved in a loyalty system with the farmers that supply fresh milk. It progressively increased the number of regular suppliers (from 9 villages in 2001 to 12 in 2002 and 15 in 2003).

DIEYE (2003) reports that the input transport cost ranges from 10.8 to 29 FCFA per litre. We know that the villages where the milk is collected are between 7.39 to 18 km from Kolda (DIEYE, 2006 and DIA, 2002). We can reasonably assume that the highest transport cost (29 FCFA) corresponds to the largest distance (18 km) and the smallest cost (10.8 FCFA) to the smallest distance (7.39 km). From that, we assume that input transport cost is approximately 1.5 F CFA per litre per km, hence $\tau = 1.5$ FCFA.

In 2001, the price paid to the farmers was 200 F CFA per litre during the wet season and 245 F CFA during the dry season (DIEYE, 2003) while the market price ranges between 350 and 450 CFA (DIEYE, 2003 and DIAO et al., 2002). From that we estimate that the price paid to the farmer is around 55% of the market price, so that we use $\psi = 0.55$.

From the income statement of Le Fermier (DIEYE, 2003), we calculate that the operating cost (including the costs of sugar, sachets, gas, electricity as well as the output transport cost) is 119 FCFA per litre. Input transport from the first village (located at 7.39 km) to the plant is independent of the size of the collection area, such that we include this cost (1.5 FCFA/km$\times$7.39 km=11.085

FCFA) in the operating cost. Thus, we use $c = 130$ FCFA. DIEYE (2003) reports that in 2001 55 000 liters of milk were collected from 9 villages which are dispersed over a distance of 6km. Hence we have $s = 25$.

BAKHOUM (2006) estimated the total investment cost between 8 and 10 million CFA Francs (CFAF). If we consider that the initial distance over which the milk was collected (6km) was the optimal entry size for the collection area we obtain that $I = 8.96 \times 10^6$ and $\kappa = 173\,485$. Based on these parameter values, the results are given in Table 2.

Table 2: Le Fermier

p^*=474.808	β_1= 1.30657
$\hat{R}(p^*)$=6	A_1 = 1.35114×10^7
$\hat{R}(p)$=$-43.6819 + 0.104636p$	$B_1(\hat{R}(p^*))$=1.33573×10^7
$\hat{R}(450)$=3.40422	$A_1(p^*)^{\beta_1}$ = 4.24386×10^{10}
$\hat{R}(500)$=8.636	$B_1(\hat{R}(p^*))(p^*)^{\beta_1} = 4.19546 \times 10^{10}$
	$g(p^*, R^*) = 4.9396 \times 10^8$

Parameter values: $I = 8.96 \times 10^6$, $\kappa = 173\,485$, $\psi = 0.55$, $c = 130$, $s = 25$, $\tau = 1.5$, $\alpha = 0.0001$, $\sigma = 0.01$ and $\rho = 0.00015$.

If the observed initial size of 6km is the optimal size, then the implied parameter values lead to the conclusion that the option to invest should be exercised when the output market price reaches $p^* = 474.808$. We have noted that the actual market price in 2001 was between 350 and 450 F CFA. Hence we can conclude from this that the entry was not done at the optimal price. Dia reports that for 2002 the highest price was 500FCFA/litre. Our calibration shows that, if the price were 500 F CFA, the optimal size would be $\hat{R}(500)$=8.636. In reality in 2002 the observed size was 10km.

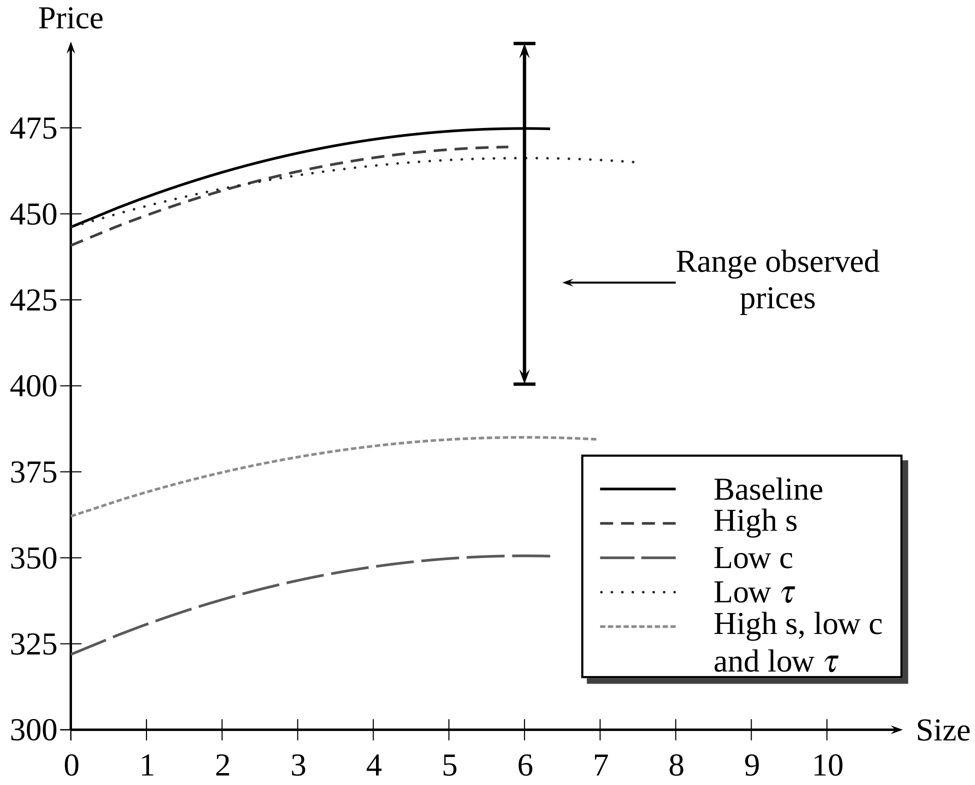
As we don't know the exact values of I and κ we check whether there are values for these two variables such that entry at the observed prices could be profitable. The total cost (TC) is given by $TC = I + \kappa \times InitialR$. Since the initial R=6, we can write κ as $\kappa = (TC - I)/6$. By letting I vary between 0 and TC we obtain all possible values for κ. Hence we obtain a continuum of combinations of values for I and κ. For each combination we can compute the optimal entry price p^* and optimal entry size R^*. The results are presented in Figure 2.

This shows that there are values of I and κ such that entry would be profitable, but only at the highest observed prices. Figure 2 also shows the effect of an increase in the supply s by 5 liters, a decrease of the cost of production c by 30% or a decrease in the transport cost τ by 30%. This shows us that even a large increase in the supply of milk in every location or a decrease of 30% is not sufficient to guarantee that entry at the observed prices is the correct decision. It would have been a correct decision at those prices if the unit cost were 30% lower. Finally, Figure 2 also shows an example of changes in parameter values which would lead to an optimal entry for all observed prices. This would be the case if, for example, supply s increased by one litre, and both c and τ would decrease by 20%.

4.2 La Laiterie du Berger

"La Laiterie du Berger" (LdB) has been producing dairy products in Richard-Toll (Northern Senegal) since 2006. It buys fresh milk from farmers dispersed on an area with a range of 50 km around

Figure 2: Le Fermier



Parameter values Baseline: $\rho = 0.00015$, $\tau = 1.5$, $\psi = 0.55$, $c = 130$, $s = 25$, $\alpha = 0.0001$, $\sigma = 0.01$; Comparative statics: $High\, s = 30$, $Low\, c = 0.7c$, $Low\, \tau = 0.7\tau$; $(High\, s,\, low\, c\, ,\, low\, \tau) = (26\, , 0.8c\, ,\, 0.8\tau)$

the plant. We establish the threshold price p^* at which the project should have been installed, as well as the optimal size of the collection area that should have been established.

Corniaux (2012) reports that the cost of transporting is 100 F CFA per litre. Cesaro (2009) reports that the length of a circuit is 100km, hence we set $\tau = 1$. We use the results of a study conducted by ABC Consulting regarding the costs associated with transforming milk in Sénégal. According to them the transformation at the plant (which includes pasteurization, packaging, etc.) costs 8 716 000 F CFA for 35 000 liters, or 183,67FCFA/litre. The output transport cost is estimated from the data of the firm Nestlé that was previously operating in Senegal, as LdB uses the same kind of transport devices, that is a refrigerated truck. DIEYE (2006) estimated that Nestlé's transport costs were 135 F CFA per litre from the collection area of Dahra to the consumption center of Dakar at 265 km, that is 0.5 F CFA per litre and per kilometer. As the LdB mainly sells its products at Dakar at 365 km from Richard-Toll, we estimate output transport costs at 182.5 F CFA per litre. The operating cost is calculated as the sum of transformation cost, cleaning and testing cost and output transport cost, such that $c = 366$. Cesaro (2009) reports that the total quantity of milk collected per day is between 1600 liters and 2000 liters, depending on the season. Since the length of the circuit is 100, we set $s = 16$.

Duteurtre (2006) reports that the market price for the products is between 750 and 1000 FCFA, depending on the volume of the package, while each farmer received 200 F CFA per litre for the milk he provides to the LdB, that is, between 20% and 26.7% of the market price. From that, we use

$\psi = 0.24$. This is lower than for the farmers contracting with Le Fermier because LdB's providers are more isolated than Le Fermiers' one. Indeed, Kolda is a urban center where fresh milk can be sold directly to the consumers, while Richard-Toll is much smaller. The closest urban center from Richard-Toll is Saint-Louis located at 120 km from the LdB.

The French Agency for Development reports that the initial investment for the LdB was 1 100 000 euros, that is 7 216 000 000 FCFA. Cesaro (2009) reports that the range of the collection circuits increased by 10km in 2008. Given that in 2009 the collection circuits had a length of 100km we assume that the initial length was 80km. We proceed as in the case of Le Fermier: assuming that the observed initial circuit length is the optimal size we compute the implied values of I and κ from the observed total initial cost. Using the values for these parameters we compute the corresponding optimal initial entry price and compare this price to the market prices observed at the time of the entry. In the case of the Ldb we have to take into account the fact that there are two collection circuits of the same length. Hence we have that $7216000000 = I + 2\times\kappa\times R^*$. From this we arrive at $I = 6.79\times 10^8$ and $\kappa = 262727$.

Parameters of the milk price distribution process are calculated similarly to what has been done in the case of Le Fermier: $\alpha = 0.0001$ and $\sigma = 0.01$. As before, we assume $\rho = 0.00015$.

Table 3: La laiterie du Berger

p^*=996.831	β_1= 1.30657
$\hat{R}(p^*)$=80	A_1 = 1.71856×10^7
$\hat{R}(p)$=$-184.237+0.265077p$	$B_1(\hat{R}(p^*))$=1.54166×10^7
$\hat{R}(1000)$=80.8402	$A_1(p^*)^{\beta_1}$ = 1.42258×10^{11}
	$B_1(\hat{R}(p^*))(p^*)^{\beta_1} = 1.27614\times 10^{11}$
	$g(p^*,R^*) = 1.53437\times 10^{10}$

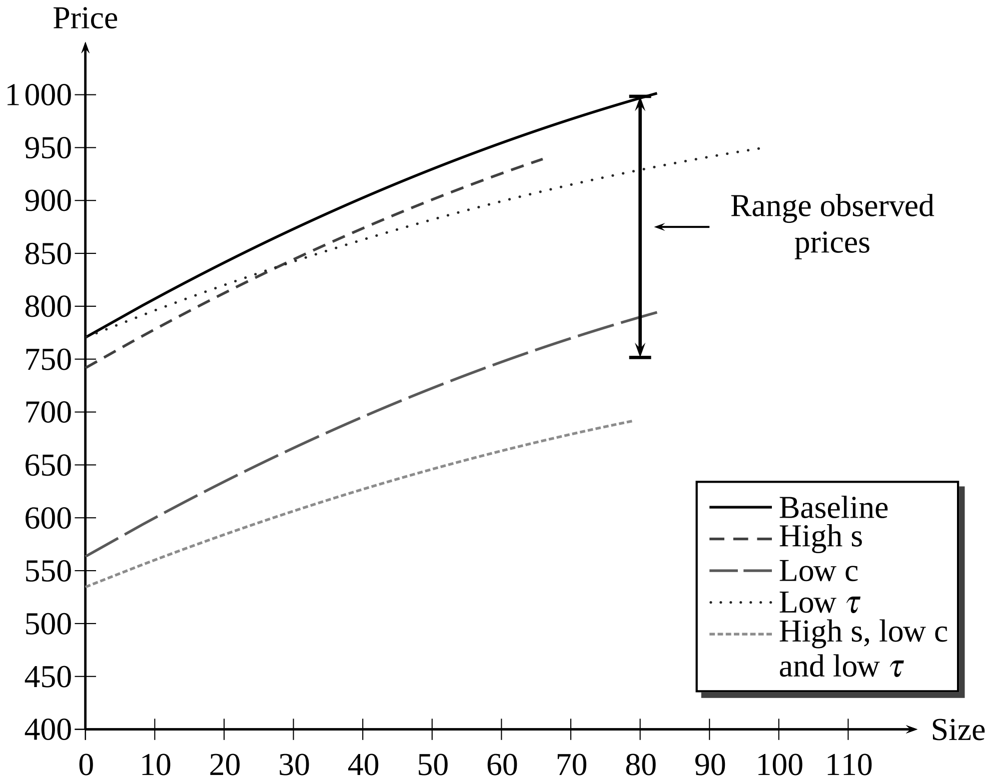
Parameter values: $I = 6.79\times 10^8$, $\kappa = 262727$, $\psi = 0.24$, $c = 366$, $s = 16$, $\tau = 1$, $\alpha = 0.0001$, $\sigma = 0.01$ and $\rho = 0.00015$.

Table 3 summarizes the main results. Assuming that the observed entry size of 80km is the optimal entry size, the parameter values imply that the optimal entry price is $p^* = 996.831$. As already mentioned, the observed market prices at that time were between 750 and 1000 FCFA, meaning that most of the observed prices for the output were below the investment threshold.

Here again, as it was the case for Le Fermier, we cannot be sure that the observed entry size is the optimal entry size. Hence here also we do the exercise for computing for all possible values of I and κ the corresponding optimal entry price p^* and optimal entry size R^*. The results are represented in Figure 3.

Our results show that for the observed prices it is impossible to find a combination of values for I and κ such that entry would be optimal at any of the observed prices. As in the case of Le Fermier, we represent the outcome if there were an increase in the supply s to 25, or a decrease in cost c of 30%, or a decrease in transport cost τ of 30%. None of these changes separately would have guaranteed that the entry at the observed prices would have been optimal. As Figure 3 illustrates a combination of, for example, a supply s of 25 with a decrease in both cost c and τ of 30% would be required to guarantee that the entry is optimal for all observed prices.

Figure 3: La Laiterie du Berger



Parameter values Baseline: $\rho = 0.00015$, $\tau = 1$, $\psi = 0.2$, $c = 366$, $s = 16$, $\alpha = 0.0001$, $\sigma = 0.01$; Comparative statics: $High\, s = 25$, $Low\, c = 0.7c$, $Low\, \tau = 0.7\tau$; $(High\, s,\, low\, c\,,\, low\, \tau) = (25\,, 0.7c\,,\, 0.7\tau)$

5 Conclusions

In this paper, we study the investment decisions of an intermediary who buys an input from geographically dispersed farmers and who sells this transformed input on a market characterized by price volatility. Due to the irreversible nature of the investment and to the uncertainty linked to price volatility, we use real options to determine at what price it is optimal for the intermediary to invest as well as the number of farmers that should be included in the collection area, both initially and in future periods.

Higher volatility is shown to postpone the initial investment, while the initial size is not affected by this factor. Hence, under higher uncertainty, the intermediary postpones the investment until the market price reaches a threshold sufficient for the intermediary to invest in a collection area of the same size. From the farmers' point of view, higher uncertainty thus means that while the same number of farmers can initially benefit from the intermediary's entry, this entry takes place later.

Furthermore, our model shows that the intermediary's entry is delayed by large investment costs, large expansion costs as well as high transport costs. In the context of developing countries, high transport costs crucially explain farmers' low participation to the market, but also tend to decrease intermediaries' entry. The State and/or donors may want to provide aid to ensure a faster entry of intermediaries which contract with a large number of farmers. Our analysis shows that providing help to reduce initial investment costs, transport costs or expansion costs will ensure a faster entry of intermediaries. Our analysis shows that an intervention that reduces the initial investment cost helps the intermediary to enter more rapidly (that is, when the market price is lower) but with a smaller collection area. Hence there is a trade-off between faster entry and smaller collection area. We also show that an intervention on transport costs or expansion costs does not lead to such a trade-off. With lower transport costs, there will be a faster entry of the intermediary with a larger collection area. However, it might be difficult or impossible to reduce transport costs for either farmers and/or intermediaries. In that case, an intervention that reduces the expansion costs is shown to reduce the price threshold for entry without reducing the optimal collection area for the intermediary.

We apply our theoretical model to two case studies in the milk sector in Senegal. The results show that the milk processing unit “Le Fermier” implanted in Kolda (Southern Senegal) since 1997 results indeed from a profitable investment decision. The actual evolution of the number of suppliers involved corresponds to the predictions of our model. Regarding “La Laiterie du Berger”, established in Richard-Toll (Northern Senegal) in 2006, the profitability is less obvious. It is likely that the initial investment would not have been profitable without the aid of donors.

The model developed here gives potential avenues for future research. We considered that the intermediary is unable to decrease the number of farmers included if they had been part of the collection area. This means that it takes its investment decisions knowing that its operating profit can become negative if the output price becomes too small. A possible extension of our model consists in the inclusion of an option to decrease the size, or at least to suspend the operation at the remote locations should the output price fall. An alternative is to consider the option to suspend the whole operation, if the operating profit becomes negative. Including such options should decrease the threshold for initial investment by offering a (not costless) protection against negative profit. Next, the accuracy of the model could be improved by considering other diffusion processes for the market price than a geometric Brownian motion. Notably, combining a jump (Poisson) process with the geometric Brownian motion would more closely represent the shocks that dramatically decrease the price.

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A Appendix A

We have:

$$V(p,R) \approx \pi(p,R)dt + E\left(V(p+dp,R)e^{-\rho dt}\right)$$

Multiplying by $e^{\rho dt}$ and subtracting $V(p,R)$ both sides, we have:

$$(e^{\rho dt}-1)V(p,R) \approx e^{\rho dt}\pi(p,R)dt + E\left(V(p+dp,R)-V(p,R)\right)$$

Using the following Taylor-MacLaurin first-order approximations: $e^{\rho dt} \approx 1+\rho dt$ and $e^{\rho dt}dt - 1 \approx \rho dt$, the above equation may be written as:

$$\rho dtV(p,R) \approx (1+\rho dt)\pi(p,R)dt + E\left(V(p+dp,R)-V(p,R)\right) \tag{A.1}$$

The term $dV \equiv V(p+dp,R)-V(p,R)$ can be expanded by Ito's lemma:

$$dV = V_p dp + \frac{1}{2}V_{pp}(dp)^2 \tag{A.2}$$

where $V_p \equiv \partial V(p,R)/\partial p$ and $V_{pp} \equiv \partial^2 V(p,R)/\partial p^2$ and where higher order terms have been dropped.

Substituting (2.1) in (A.2) gives:

$$dV = V_p(\alpha pdt + \sigma pdz) + \frac{1}{2}V_{pp}(\alpha pdt + \sigma pdz)^2$$

$$\Leftrightarrow dV = \alpha pV_p dt + \sigma pV_p dz + \frac{1}{2}V_{pp}(\alpha^2 p^2 (dt)^2 + \sigma^2 p^2 (dz)^2 + 2\alpha\sigma p^2 dtdz) \tag{A.3}$$

As dz is the increment of a Wiener process with $E[dz]=0$, multiplication rules of Ito's lemma tell us that $E[(dz)^2]=dt$, $E[dzdt]=E[dtdz]=0$ and $E[(dt)^2]=0$. As αpdt is deterministic, we know that $E[\alpha pdt]=\alpha pdt$. Thus, taking the expectation of (A.3) gives:

$$E[dV] = \alpha pV_p dt + \frac{1}{2}V_{pp}\sigma^2 p^2 dt. \tag{A.4}$$

Replacing $E[dV]$ by (A.4) in (A.1), dividing by dt and taking the limit for $dt \to 0$, we get:

$$\rho V(p,R) = \pi(p,R) + \alpha pV_p + \frac{1}{2}V_{pp}\sigma^2 p^2$$

From that, we have the differential equation:

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 V(p,R)}{\partial p^2} + \alpha p \frac{\partial V(p,R)}{\partial p} - \rho V(p,R) + \pi(p,R) = 0$$

B Appendix B

We have:

$$F(p) \approx e^{-\rho dt} E[F(p+dp)]$$

Expanding the discount factor we have

$$F(p) \approx (1-\rho dt) E[F(p+dp)]$$

This can be written as

$$\begin{aligned} F(p) &\approx (1-\rho dt) E[dF + F(p)] \\ &\approx (1-\rho dt)(E[dF] + F(p)) \end{aligned} \tag{B.1}$$

where $dF \equiv F(p+dp) - F(p)$. Applying Ito's Lemma to dF, dropping higher order terms and taking the expectation we have

$$E(dF) = \alpha p F_p dt + \frac{1}{2}\sigma^2 p^2 F_{pp} dt \tag{B.2}$$

Plugging (B.2) in (B.1) and taking the limit $dt \to 0$ we obtain

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 F(p)}{\partial p^2} + \alpha p \frac{\partial F(p)}{\partial p} - \rho F(p) = 0$$

C Appendix C

Finding the solution to equation (2.2) is straightforward. Equation (2.2) is a non-homogeneous second-order Euler-Cauchy differential equation. The solution to equation (2.2) is given by the sum of the solution to the homogeneous part of the equation and the particular solution of the full equation.

The solution of the homogeneous part of the equation can be expressed as a linear combination of any two independent solutions:

$$V_h(p) = B_1 p^{\beta_1} + B_2 p^{\beta_2}$$

where $\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}}$ and $\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}}$ are respectively the positive and the negative root of the quadratic characteristic equation: $\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - \rho = 0$.

To find the particular solution to the nonhomogeneous equation we proceed as follows. Given that $y_1(p) = p^{\beta_1}$ and $y_2(p) = p^{\beta_2}$ are solutions to the homogeneous part of the equation, the particular solution can be written as:

$$y_{part}(p) = c_1(p) y_1(p) + c_2(p) y_2(p)$$

where

$$
\begin{aligned}
c_1(p) &= -\int_{p_1}^{p} \frac{y_2(\eta)\,r(\eta)}{W(\eta)} d\eta \\
c_2(p) &= \int_{p_2}^{p} \frac{y_1(\eta)\,r(\eta)}{W(\eta)} d\eta
\end{aligned}
$$

with $r(\eta) = \pi(\eta, R) / \frac{1}{2}\sigma^2\eta^2$ and $W(\eta)$ is the Wronksian and given by $y_1 y_2' - y_2 y_1' = \eta^{\beta_1}\beta_2\eta^{\beta_2-1} - \eta^{\beta_2}\beta_1\eta^{\beta_1-1} = (\beta_2 - \beta_1)\eta^{\beta_1+\beta_2-1}$. The integration limits p_1 and p_2 are given by the boundary conditions. Since the boundary conditions imply that $c_1(\infty) = 0$ and $c_2(0) = 0$ this leads to

$$
y_{part}(p) = \frac{2}{(\beta_1 - \beta_2)\,\sigma^2} \left[y_2(p) \int_0^p \frac{y_1(\eta) r(\eta)}{W(\eta)} d\eta + y_1(p) \int_p^\infty \frac{y_2(\eta) r(\eta)}{W(\eta)} d\eta \right]
$$

Plugging the different elements in the expression above we have

$$
g(p,R) \equiv y_{part}(p) = \frac{2}{(\beta_1 - \beta_2)\,\sigma^2} \left[p^{\beta_2} \int_0^p \frac{\pi(\eta,R)}{\eta^{\beta_2+1}} d\eta + p^{\beta_1} \int_p^\infty \frac{\pi(\eta,R)}{\eta^{\beta_1+1}} d\eta \right] \tag{C.1}
$$

Furthermore we have

$$
\begin{aligned}
g_R(p,R) &= \frac{2}{(\beta_1 - \beta_2)\,\sigma^2} \left[p^{\beta_2} \int_0^p \frac{\pi_R(\eta,R)}{\eta^{\beta_2+1}} d\eta + p^{\beta_1} \int_p^\infty \frac{\pi_R(\eta,R)}{\eta^{\beta_1+1}} d\eta \right] > 0 \\
g_{RR}(p,R) &= \frac{2}{(\beta_1 - \beta_2)\,\sigma^2} \left[p^{\beta_2} \int_0^p \frac{\pi_{RR}(\eta,R)}{\eta^{\beta_2+1}} d\eta + p^{\beta_1} \int_p^\infty \frac{\pi_{RR}(\eta,R)}{\eta^{\beta_1+1}} d\eta \right] < 0
\end{aligned}
$$

By differentiating (C.1) with respect to p and integrating by parts, we obtain

$$
g_p(p,R) = \frac{2}{(\beta_1 - \beta_2)\,\sigma^2} \left[p^{\beta_2-1} \int_0^p \frac{\pi_p(\eta,R)}{\eta^{\beta_2}} d\eta + p^{\beta_1-1} \int_p^\infty \frac{\pi_p(\eta,R)}{\eta^{\beta_1}} d\eta \right] > 0 \tag{C.2}
$$

By differentiating (C.2) with respect to p and integrating by parts, we obtain

$$
g_{pp}(p,R) = \frac{2}{(\beta_1 - \beta_2)\,\sigma^2} \left[p^{\beta_2-2} \int_0^p \frac{\pi_{pp}(\eta,R)}{\eta^{\beta_2-1}} d\eta + p^{\beta_1-2} \int_p^\infty \frac{\pi_{pp}(\eta,R)}{\eta^{\beta_1-1}} d\eta \right] \geq 0
$$

Similarly we establish that $g_{pR} > 0$ and $g_{Rpp} \geq 0$.

D Appendix D

We start by showing that $\hat{p}(R)$ exists. Rewriting the expression (2.10) as

$$
f_1(\hat{p}(R), R) \equiv \hat{p}(R) \frac{\partial^2 g(\hat{p}(R), R)}{\partial R \partial p} - \beta_1 \frac{\partial g(\hat{p}(R), R)}{\partial R} + \beta_1 \kappa = 0
$$

We have

$$
\begin{aligned}
f_{1R} &= \hat{p}(R) \frac{\partial^3 g(\hat{p}(R), R)}{\partial R \partial p \partial R} - \beta_1 \frac{\partial^2 g(\hat{p}(R), R)}{(\partial R)^2} > 0 \\
f_{1p} &= \frac{\partial^2 g(\hat{p}(R), R)}{\partial R \partial p} + \hat{p}(R) \frac{\partial^3 g(\hat{p}(R), R)}{\partial R \partial p \partial p} - \beta_1 \frac{\partial^2 g(\hat{p}(R), R)}{\partial R \partial p} < 0
\end{aligned}
$$

Since $f_1(0,R)>0$ and $f_{1p}<0$ we have that for every R there is a price p such that the above expression is verified. We call this price $\hat{p}(R)$. $\hat{p}(R)$ is increasing since $d\hat{p}/dR=-(f_{1R})/f_{1p}>0$. Furthermore, since $f_{1\kappa}=\beta_1>0$, we have that $d\hat{p}/d\kappa=-(f_{1\kappa})/f_{1p}>0$. Using the fact that $\partial\beta_1/\partial\sigma<0$ and that $f_{1\sigma}=-\frac{\partial\beta_1}{\partial\sigma}\left(\frac{\partial g(\hat{p}(R),R)}{\partial R}-\kappa\right)>0$, we have that $d\hat{p}/d\sigma=-(f_{1\sigma})/f_{1p}>0$.

E Appendix E

We start by showing that $p^*(R)$ exists. We can rewrite the expression (2.13) as

$$f_2(p^*(R),R)\equiv\beta_1\kappa R+\beta_1 I+p^*(R)\frac{\partial g\left(p^*(R),R\right)}{\partial p}-\beta_1 g\left(p^*(R),R\right)=0$$

We have

$$\begin{aligned}
f_{2R}&=\beta_1\kappa+p^*(R)\frac{\partial^2 g\left(p^*(R),R\right)}{\partial p\partial R}-\beta_1\frac{\partial g\left(p^*(R),R\right)}{\partial R}\\
f_{2p}&=\frac{\partial g\left(p^*(R),R\right)}{\partial p}+p^*(R)\frac{\partial^2 g\left(p^*(R),R\right)}{(\partial p)^2}-\beta_1\frac{\partial g\left(p^*(R),R\right)}{\partial p}<0
\end{aligned}$$

Since $f_2(0,R)>0$ and $f_{2p}<0$ we have that for every R there is a price p such that the above expression is verified. We call this price $p^*(R)$. The sign of f_{2R} depends on the value of R: for small values, we have a negative slope, while for large values we have a positive slope. Furthermore, since $f_{2\kappa}=\beta_1 R>0$, we have that $dp^*(R)/d\kappa=-(f_{2\kappa})/f_{2p}>0$. Using the fact that $\partial\beta_1/\partial\sigma<0$ and that $f_{2\sigma}=-\frac{\partial\beta_1}{\partial\sigma}\left(\kappa R+I-g\left(p^*(R),R\right)\right)>0$, we have that $dp^*(R)/d\sigma=-(f_{2\sigma})/f_{2p}>0$. Finally, since $f_{2I}=\beta_1>0$, we have that $dp^*(R)/dI=-(f_{2I})/f_{2p}>0$.

F Appendix F

p^* and R^* are the solution of the system of equations given by (2.10) and (2.13) which we write as $f_1(p,R)=0$ and $f_2(p,R)=0$. There is an intersection of the two curves since $f_{2p}(p,R)=f_1(p,R)$. The intersection is unique since, evaluated at the solution, we have:

$$\begin{vmatrix} f_{1p} & f_{1R}\\ f_{2p} & f_{2R}\end{vmatrix}=f_{1p}f_{2R}-f_{1R}f_{2p}=-f_{1R}f_{2p}>0$$

G Appendix G

Transforming the equations (3.4) and (3.6) into

$$\begin{aligned}
F &\equiv \frac{\rho}{2\tau}\left(\frac{\beta_1-1}{\beta_1}\frac{(1-\psi)\,p}{\rho-\alpha}-\frac{\kappa}{s}-\frac{c}{\rho}\right)-R=0\\
G &\equiv \frac{\beta_1}{\beta_1-1}\frac{\rho-\alpha}{(1-\psi)}\left(\frac{\kappa}{s}+\frac{c+\tau R}{\rho}+\frac{I}{sR}\right)-p=0
\end{aligned}$$

Total differential of each equation for a variable x

$$\begin{aligned} F_p dp + F_R dR + F_x dx &= 0 \\ G_p dp + G_R dR + G_x dx &= 0 \end{aligned}$$

Dividing by dx and rearranging the terms

$$\begin{aligned} F_p \frac{dp}{dx} + F_R \frac{dR}{dx} &= -F_x \\ G_p \frac{dp}{dx} + G_R \frac{dR}{dx} &= -G_x \end{aligned}$$

In matrix form

$$\begin{bmatrix} F_p & F_R \\ G_p & G_R \end{bmatrix} \begin{bmatrix} \frac{dp}{dx} \\ \frac{dR}{dx} \end{bmatrix} = \begin{bmatrix} -F_x \\ -G_x \end{bmatrix}$$

Hence

$$\begin{aligned} \begin{bmatrix} \frac{dp}{dx} \\ \frac{dR}{dx} \end{bmatrix} &= \begin{bmatrix} F_p & F_R \\ G_p & G_R \end{bmatrix}^{-1} \begin{bmatrix} -F_x \\ -G_x \end{bmatrix} = \frac{1}{F_p G_R - F_R G_p} \begin{bmatrix} G_R & -F_R \\ -G_p & F_p \end{bmatrix} \begin{bmatrix} -F_x \\ -G_x \end{bmatrix} \\ &= \frac{1}{F_p G_R - F_R G_p} \begin{bmatrix} -G_R F_x + F_R G_x \\ G_p F_x - F_p G_x \end{bmatrix} \end{aligned}$$

At the equilibrium we have $G_R = 0$ which leads to

$$\begin{bmatrix} \frac{dp}{dx} \\ \frac{dR}{dx} \end{bmatrix} = \frac{1}{-F_R G_p} \begin{bmatrix} F_R G_x \\ G_p F_x - F_p G_x \end{bmatrix} = \begin{bmatrix} -\frac{G_x}{G_p} \\ -\frac{F_x}{F_R} + \left(-\frac{F_p}{F_R}\right)\left(-\frac{G_x}{G_p}\right) \end{bmatrix}$$

Hence we have $\frac{dp^*}{dx} = -\frac{G_x}{G_p}$ meaning that the effect of x on the entry price depends only on its effect on the entry price threshold. We also have

$$\frac{dR^*}{dx} = -\frac{F_x}{F_R} + \left(-\frac{F_p}{F_R}\right)\left(-\frac{G_x}{G_p}\right) = \frac{\partial \hat{R}(p^*, x)}{\partial x} + \frac{\partial \hat{R}(p^*, x)}{\partial p} \frac{dp^*}{dx}$$

In addition, we have the following results for the effects of the parameters on the entry price and optimal initial size:

Table 4: Expressions for the effect of parameters on p^* and R^*

$\frac{dp^*}{d\sigma} = \left(\frac{\kappa}{s} + \frac{c}{\rho} + 2\sqrt{\frac{\tau I}{s\rho}}\right)\frac{\rho-\alpha}{1-\psi}\frac{-1}{(\beta_1-1)^2}\frac{\partial\beta_1}{\partial\sigma} > 0$	$\frac{dR^*}{d\sigma} = 0$
$\frac{dp^*}{dI} = \frac{\beta_1(\rho-\alpha)}{(\beta_1-1)(1-\psi)}\sqrt{\frac{\tau}{s\rho I}} > 0$	$\frac{dR^*}{dI} = \frac{1}{2I}\sqrt{\frac{\rho I}{\tau s}} > 0$
$\frac{dp^*}{d\kappa} = \frac{\beta_1(\rho-\alpha)}{(\beta_1-1)(1-\psi)}\frac{1}{s} > 0$	$\frac{dR^*}{d\kappa} = 0$
$\frac{dp^*}{d\tau} = \frac{\beta_1(\rho-\alpha)}{(\beta_1-1)(1-\psi)}\sqrt{\frac{I}{\tau s\rho}} > 0$	$\frac{dR^*}{d\tau} = \frac{-1}{2\tau}\sqrt{\frac{\rho I}{\tau s}} < 0$
$\frac{dp^*}{d\psi} = \frac{\beta_1(\rho-\alpha)}{(\beta_1-1)(1-\psi)^2}\left(\frac{\kappa}{s} + \frac{c}{\rho} + 2\sqrt{\frac{\tau I}{s\rho}}\right) > 0$	$\frac{dR^*}{d\psi} = 0$
$\frac{dp^*}{ds} = \frac{\beta_1(\rho-\alpha)}{(\beta_1-1)(1-\psi)^2}\left(\frac{-\kappa}{s^2} + \frac{-1}{s^2}\sqrt{\frac{\tau Is}{\rho}}\right) < 0$	$\frac{dR^*}{ds} = \frac{-1}{2s}\sqrt{\frac{\rho I}{\tau s}} < 0$