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How to license a downstream technology when upstream firms are capacity constrained?*

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Abstract. In this paper, we study the relationship between capacity constraints and licensing strategies. To do so, we focus on the licensing strategy of an outside innovator who licenses a process innovation to the downstream sector of a vertical Cournot oligopoly. Downstream firms source an essential production factor from a capacity constrained upstream sector. In this setting, we show that the innovator optimally licenses large innovations via per-unit royalty contracts and small innovations via fixed fee contracts. Moreover, an increase in the strength of the capacity constraints makes it more likely that the optimal licensing contract includes a strictly positive per-unit royalty rate. As a final point, we discuss the relationship between capacity constraints and the social optimality of the innovator's licensing strategy as measured by aggregate welfare or the diffusion of the innovation on the downstream market.

Keywords capacity constraints, licensing contracts, vertical Cournot oligopoly.
JEL classification D43, L13, O31, O34.

1 Introduction

"On the one hand, demand for raw material continues to grow unabated: [factors such as] the continuous emergence of new technologies and applications place ever greater demand on natural resources. On the other hand, the supply of a wide range of natural resources [...] face increasing constraints [...]". (KPMG Advisory N.V. (2012, p.1))

Raw materials, such as minerals and metals, are the building blocks of modern technologies, economies and societies. "*Without minerals, industrial society and modern technology would be inconceivable. If fossil fuels are the proverbial lifeblood of the global economy, then minerals are certainly its bone marrow*" (Kooroshy et al. (2009, p.17)). Nevertheless, over the past years, the availability of certain raw materials has become an issue of growing concern among business

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leaders and policy makers alike. For example, a 2011 study by PwC reveals that almost 70% of business executives consider minerals and metals scarcity an important topic. Moreover, the supply risk of the latter materials is expected to increase over the next five years, potentially resulting in supply instabilities and disruptions with consequences for the entire supply chain. In the same vein, initiatives developed by the European Commission (2008; Raw Materials Initiative) and the U.S. Department for Energy (2010; Critical Materials Strategy) reflect the importance of the issue.

Contrary to common belief, the reasons for the scarcity of certain raw materials are mainly rooted in a (temporary) imbalance of demand and supply, rather than in a depletion of natural resources (Angerer et al. (2009), Kooroshy et al. (2010)). There are many factors that contribute to this imbalance or *economic scarcity* of raw materials. For example, an important number of minerals and metals derive their scarcity from the fact that their occurrence in economically mineable deposits is limited and/or restricted to a limited number of countries.[1] Moreover, raw materials are frequently the subject of trade barriers and/or geopolitical conflicts.[2]

The primary reason for a scarcity of technology relevant minerals and metals appears, however, to lie in an unprecedented increase in their demand over the past decades. Here, one main driver is technological progress (Angerer et al. (2009), Kooroshy et al. (2010), Speirs et al. (2014)). As such, the past years have seen the rapid emergence of high technology products and production processes which heavily rely on minerals and metals. The threat of a demand-side scarcity of raw materials is amplified by the fact that the use of a particular raw material is not limited to a single sector. An example are, for instance, Gallium and Indium which are essential for emerging technologies in fields such as clean energy supply (thin film photovoltaics), information and communication (high performance chips, flat screens) and lighting (light emitting diodes). Angerer et al. (2009) estimate that by 2030 the technology-induced demand for Gallium and Indium will be approximately 6 and 3.3 times higher than their world wide production in 2006. Another important point is the fact that many technology-relevant metals are essentially by-products of other metals. Their supply is therefore restricted by the supply of their carrier metal, which may place a strict upper limit on their availability (*structural scarcity*) (Hagelüken and Meskers (2010)).

Technological progress clearly is not restricted to the outcome of internal research and development (R&D) activities. Instead, firms, next to or instead of their own internal R&D activities, increasingly rely on licensing agreements to access external knowledge sources. This trend is reflected in the data which indicates that technology licensing activities have increased over the past two decades. For example, Sheehan et al. (2004) find that almost 60% of the surveyed

[1] For instance, Brazil produces 90% of Niobium, South-Africa 79% of Rhodium and 77% of Platinum, China 95% of Rare Earth Elements, 87% of Antimony and 84% of Tungsten (European Commission (2014)). The latter elements find application in, among other things, fuel cells, energy-efficient lighting systems, emission prevention and purification technology, super alloys, catalytic converters as well as in fiber- and flat-panel glass.

[2] In this context, a well-known example are the export restrictions on Rare Earth Elements that were introduced by China in 2010. Rare Earth Elements form a group of seventeen chemical elements and are essential production factors for most recent technologies. This is particularly true for green technologies such as energy-efficient lighting systems, solar panels or high performance batteries. At present, China is the world's leading exporter of REEs. Its major importers (Europe, Japan and the United States) source more than 90 % of their input requirements from the country.

firms in Europe, the United States, and Asia-Pacific report increased licensing activities during the 1990s. Similarly, Zuniga and Guellec (2009) state that 45% of European firms in their sample report increased licensing revenues or activities between 2003 and 2006. On the aggregate, Robbins (2006) estimates that the licensing of industrial processes amounted to $66 billion in 2002 in the United States (as compared to $27.4 billion in 1995), indicating a global market for technology of around $100 billion (Arora and Gambardella (2010)).

In this paper, we study such licensing agreements, taking into account that an input factor, which is essential for production in the licensed industry, may be scarce. We especially focus on the question of how the presence of a scarce input factor alters the incentives to transfer and adopt process technologies as well as the design of the optimal licensing contract.

To address these questions, we adopt the following modeling framework. There is one actor, the innovator, who holds a patent to a process technology. The innovator is unable to commercialize the technology him-/herself[3] and instead licenses it to some manufacturing industry by means of a fixed fee or per-unit royalty licensing contract (in an extension we also address two-part tariffs, i.e., combinations of fixed fee and per-unit royalty contracts).[4] Firms in the manufacturing industry compete *à la Cournot* and source their input requirements from an oligopolistic upstream sector. In line with the motivation for this paper, the upstream sector may be seen as the sector of raw materials refiners. Alternatively, upstream firms may be seen as the producers of some intermediate input, which itself relies on a scarce raw material. Firms in this sector are capacity constrained and compete in quantities. In this paper, we focus on a demand-side scarcity of the input factor. That is, upstream firms' supply restrictions arise as a corollary of technological progress, which, in our framework, corresponds to the transfer of the process technology to the manufacturing sector. We focus on two particular cases: strict capacity constraints (upstream firms are unable to produce more than prior to the technology transfer) and soft capacity constraints (any production above the pre-technology transfer level is characterized by decreasing returns to scale). Strict capacity constraints correspond, for instance, to a scenario in which upstream firms operate at their capacity level prior to the technology transfer and are reluctant to invest in their capacities (prohibitively high cost, uncertainty of demand conditions) or face a significant time lag between the investment and the capacity expansion.

Our analysis shows that the scarcity of an essential production factor has important implications for the design of the optimal licensing contract as well as for its welfare performance. As such, we demonstrate that in contrast to the traditional licensing literature (among others, Kamien et al. (1992), Kamien and Tauman (1984b), Kamien and Tauman (1986)) a per-unit royalty contract may provide larger licensing revenues than a fixed fee scheme. In particular, we find that the innovator licenses large innovations via per-unit royalty agreements and small

[3] Such independent technology suppliers, or outside innovators, are responsible for an (increasingly) important share of licensing agreements. For example, Cesaroni (2003) reports that independent technology suppliers cover almost 70% of the total market for licensing in the chemical industry. Similarly, most of the technology transfer in functional design inventions or design modules in the semiconductor industry involves chipless firms (Linden and Somaya (2003)).

[4] Other licensing mechanisms that are studied in the literature are auctions. However, this type of licensing policy is less frequently observed in practice which may be explained by its high organizational costs relative to other types of licensing agreements (see also **?**).

innovations via fixed fee contracts. Moreover, an increase in the scarcity of the input factor makes it more likely that per-unit royalty contracts are the optimal licensing policy. Our paper thus provides another rationale for the empirically observed popularity of royalty licensing agreements (Bousquet et al. (1998), Macho Stadler et al. (1996), Rostoker (1983-1984), Sakakibara (2010)).[5] In an extension, we also address two-part tariffs. In the context of a vertical Cournot duopoly, we show that the intuition that underlies our main results carries over to two-part tariffs: the optimal licensing contract is likely to involve per-unit royalty payments whenever the technology offers an important cost reduction and the supply of the input factor faces serious constraints. In fact, for sufficiently strict constraints, the optimal two-part tariff may reduce to a pure per-unit royalty contract.

To discuss the welfare implications of the optimal licensing contract we focus on the relationship between capacity constraints and the social optimality of the innovator's licensing strategy as measured by aggregate economic welfare or the diffusion of the innovation in the downstream market. In this context, two observations stand out. First, depending on the size of the innovation and the strength of the capacity constraints, per-unit royalty contracts may maximize aggregate welfare. Second, there is scope for conflict between the innovator's licensing strategy and the socially optimal policy.

Our results are based on the presence of a sufficiently strong *capacity effect*, which reduces the private optimality of fixed fee agreements. The capacity effect works as follows. The process technology improves the productive efficiency of any licensed firm in the manufacturing sector (reduced marginal production costs). As such, the adoption of the technology is followed by an expansion of downstream production levels and, consequently, by an increased demand for the input factor. The upstream sector, however, is unable to fully accommodate this higher level of downstream demand. As a corollary, the innovation results in a substantial increase of the input price. The higher level of the input price then reduces downstream firms' willingness to pay for the innovation, as well as the number of licensed downstream firms (and hence equally the innovator's fixed fee licensing income). Under a per-unit royalty contract, this effect is absent. The reason is that per-unit royalty rates increase marginal production costs of licensed firms and, at the optimum, exactly offset the cost-reduction of the innovation. It follows that capacity constraints reduce the attractiveness of fixed fee relative to per-unit royalty licensing agreements.

Our paper is structured as follows. In Section 2 we introduce the general modeling framework and derive the optimal per-unit royalty and fixed fee licensing contracts. Section 3 then derives the overall optimal licensing contract by contrasting the innovator's licensing income for the two licensing policies. In Section 4 we study some welfare aspects of the privately optimal licensing contracts. In this context, we also address the relationship between capacity constraints and the

[5] There is a large theoretical literature that tries to rationalize the empirically observed popularity of per-unit royalty licensing agreements. Approaches range from the consideration of a leadership structure (Filippini (2005), Kabiraj (2004)) to the introduction of product differentiation (Bagchi and Mukherjee (2014), Erkal (2005), Kabiraj and Lee (2011)) or informational asymmetries (Beggs (1992), Gallini and Wright (1990)). Other authors rationalize the popularity of royalty licensing agreements by taking into account that the innovator may compete in the product market (inside innovator) (Kamien and Tauman (2002), Wang (1998)) or by acknowledging the fact that the number of licensing contracts offered may only take on integer values (Sen (2005)).

diffusion of process technology. Section 5 addresses two-part tariff licensing contracts. Section 6 concludes.

2 The model

2.1 Description and market stage analysis

We consider a vertical Cournot oligopoly with an upstream (m) and a downstream (n) sector. On the upstream sector, M firms are active and produce a homogeneous product. This product serves as an input for the downstream industry on a one-to-one basis. It is assumed that upstream firms move first and compete *à la Cournot*. Their individual quantities are denoted by x_i. Quantity competition upstream yields the market clearing input price w which is taken as given by the downstream firms.[6] On the downstream market, N firms produce a final homogeneous good. Downstream firms compete *à la Cournot* where q_i^α refers to the individual downstream quantities. Here $\alpha \in \{l, nl\}$ denotes the licensing status of a firm (l for *licensed* and nl for *not licensed*). The inverse demand function is of a standard form and given by $p(Q) = a - Q$ with $Q = \sum_{i=1}^{N} q_i^\alpha$. Next to the input price w, downstream firms face their constant marginal cost of production c_i^α. Intuitively, $c_i^l < c_i^{nl}$. More precisely, downstream firms have potential access to a cost-reducing technology (process innovation) which lowers their marginal production costs by some strictly positive constant θ.[7] It follows that a downstream firm produces with either $c_i^l = c - \theta$ or $c_i^{nl} = c$, depending on its licensing status. The technology is supplied by an outside innovator, who is not active in either industry sector and may transfer the technology via either a per-unit royalty or a fixed fee licensing contract.

We model the analysis as a non-cooperative game in four stages. First, depending on the type of licensing policy chosen, the innovator sets either a fixed fee or a per-unit royalty rate for all licensees. Second, downstream firms independently and simultaneously decide whether to accept or refuse the proposed contract. Those decisions divide the downstream sector in two sets of licensed and unlicensed firms. Third, once the number of licensed firms is determined, upstream firms compete in quantities which yields the market clearing input price.[8] Fourth, the input price is observed by the downstream firms, which then also compete in quantities.

[6] Our approach is a standard one in the literature of successive oligopolies (see, for instance, Ghosh and Morita (2007), Greenhut and Ohta (1979), Peitz and Reisinger (2014) or Salinger (1988)). It provides us with a tractable modeling framework in which we are able to analyze the interaction between a capacity constrained upstream sector and a downstream sector in which firms, endogenously, differ in their production efficiency. One implication of the model is that downstream firms act as price takers with respect to the input price when taking their production decisions. The downstream firms' price taking behavior may be rationalized by the assumption that the input factor is supplied to a large number of independent downstream industries so that firms in a particular industry only have a negligible effect on the input price. In our setting, in which the upstream sector may be seen as supplying raw materials, this assumptions appears to be reasonable.

[7] Our focus is on non-drastic innovations. By this, we mean that we focus on the cases in which the innovator licenses at least two firms if the innovation is sufficiently large such that any non-licensee realizes non-positive market revenues. For a standard, i.e., non-vertically separated, Cournot oligopoly this definition of a non-drastic innovation coincides with the on established by Arrow (1962).

[8] In our eyes, it is reasonable to assume that the licensing contracts are signed before the upstream firms announce the input price. The adoption of process technology is a costly and lengthy process which is typically planned over a longer time horizon. What is more, a non-trivial share of licensing agreements are signed over prospective technologies (see, e.g., Anand and Khanna (2000)).

$i \in \{1, \ldots, M\}$	Upstream firms (input supplier).
$i \in \{1, \ldots, N\}$	Downstream firms (final good producers).
x_i^k/X^k	Upstream production levels (individual/aggregate).
$q_i^{\alpha,k}/Q^k$	Downstream production levels (individual/aggregate).
$k \in [0, \infty)$	Strength of upstream capacity constraints ($k = 0/k \in (0, \infty)/k \to \infty$ corresponds to no/soft/strict capacity constraints).
$\alpha \in \{l, nl\}$	Licensing status (l=*licensed*, nl=*not licensed*).
θ	Process innovation/cost-reduction.
l^k	Number of licensing contracts.
r^k	Per-unit royalty rate.
f^k	Fixed fee.

Table 1: Parameters and variables.

On the last stage of the game, downstream firms maximize their profits $\pi_i^{n,\alpha,k} = \big(p(Q^k) - w - c_i^\alpha\big)q_i^{\alpha,k}$ with respect to $q_i^{\alpha,k}$. In the latter expression, $k \in [0, \infty)$ refers to the strength of the upstream firms' capacity constraints (we give more details below). Summing the corresponding first-order conditions for the group of licensed, l^k, and unlicensed, $N - l^k$, downstream firms, we obtain the indirect derived demand for input goods as

$$w(Q^k) = a - \frac{C}{N} - \Big(\frac{N+1}{N}\Big)Q^k \tag{1}$$

with $C = \sum_{i=1}^N c_i^\alpha = Nc - l^k\theta$. Note that $C < Nc$ for $l^k \geq 1$.

In the following, $w(Q^k)$ is correctly anticipated by the upstream firms. The cost function of a typical upstream firm is given by

$$C_i^m(x_i^k) = \begin{cases} 0 & x_i^k \leq \bar{x}, \\ \frac{k}{2}(x_i^k - \bar{x})^2 & x_i^k > \bar{x}. \end{cases} \tag{2}$$

In words, the total production costs of an upstream firm are zero up to a certain threshold, denoted by $\bar{x}$. For all units of production exceeding this threshold, i.e., for any $x_i^k - \bar{x} > 0$, an upstream firm occurs total costs of $k(x_i^k - \bar{x})^2/2$. Note that those costs are convex so that any production above $\bar{x}$ is characterized by decreasing returns to scale. The degree of convexity of the cost function is measured by k. The larger k, the stricter capacity constraints (or the scarcer the input factor) and the more costly it is to expand production above $\bar{x}$. For $k \to \infty$, upstream firms are unable to expand their production above $\bar{x}$. In line with the motivation for this paper, we focus on a particular threshold and set $\bar{x} = x_{pre}$ with x_{pre} the individual production level of an upstream firm in the pre-licensing equilibrium. This captures the idea that the scarcity of the input factor is mainly driven by technological progress (demand-side scarcity).[9]

To illustrate how capacity constraints of this form impact the market equilibrium, we first

[9] We discuss how our results depend on the choice of capacity level at the end of Section 3.

introduce a benchmark scenario in which upstream firms face no constraints to output expansion, i.e., $k = 0$. In the absence of capacity constraints, each upstream firm maximizes $\pi_i^{m,0} = w(X^0)x_i^0$ with respect to x_i^0. This yields aggregate industry output

$$X^0 = \frac{MN(a - \frac{C}{N})}{(M+1)(N+1)}. \tag{3}$$

As a consequence, the input and final good price are

$$w^0 = \frac{a - \frac{C}{N}}{M+1} \quad \text{and} \quad p^0 = \frac{a(M+N+1) + MC}{(M+1)(N+1)}. \tag{4}$$

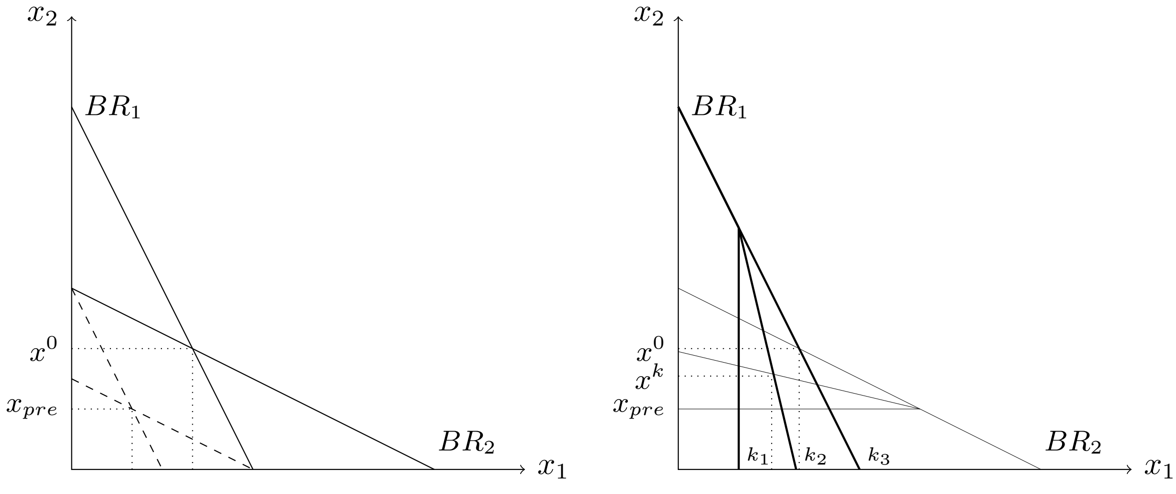
Clearly, for $C = Nc$ (all downstream firms produce at their pre-licensing marginal costs), $X^0 = Mx_{pre} = X_{pre}$, $p^0 = p_{pre}$, and $w^0 = w_{pre}$. From (3) and (4) it is further immediate that process innovations feature an expansion effect. On the aggregate, this implies that industry output expands proportionally to the decrease in downstream marginal cost of production.

Lemma 1 *As the sum of the marginal costs of the downstream industry decreases, aggregate industry output and the intermediary input price increase, whereas the final good price decreases. That is, $\partial X^0/\partial C < 0$, $\partial w^0/\partial C < 0$, while $\partial p^0/\partial C > 0$.*

The implications of the process innovation for the upstream market equilibrium, in the absence of capacity constraints, are illustrated in Figure 1a. The graph is based on a vertical Cournot duopoly ($M = N = 2$) in which upstream firms are unconstrained ($k = 0$) and the innovation is licensed to both downstream firms ($l^0 = 2$). The firms' best-response functions prior to any technology transfer are given by the dashed lines. In the pre-licensing equilibrium, each upstream firm consequently supplies an amount equal to x_{pre}. The transfer of the process technology to the downstream market leads to a parallel outward shift of the upstream firms' best response functions (to BR_1 and BR_2). As a result, both firms produce x^0 with $x^0 > x_{pre}$ in the post-licensing equilibrium.

Let us now assume that both upstream firms are capacity constrained, i.e., face total production costs as specified in (2). Figure 1b illustrates the situation for different values of k. First, for $k \to \infty$, capacity constraints become strict and upstream firms are unable to expand their production beyond the pre-licensing level. In this case, the best-response function of an upstream firm has a vertical segment. This corresponds to the part of BR_1 (respectively BR_2) where the supply of firm 2 (firm 1) is sufficiently small such that firm 1's (firm 2's) capacity constraints are binding. In the post-licensing equilibrium, both upstream firms therefore continue to produce x_{pre}. For $k \in (0, \infty)$, capacity constraints are soft. Upstream firms may produce more than x_{pre}, however, face increasingly higher costs at larger values of x_i^k. Moreover, as k increases, an expansion of production levels beyond x_{pre} becomes more and more costly; as k increases (capacity constraints tighten), $x_i^k \to x_{pre}$, while as k decreases, (capacity constraints soften) $x_i^k \to x^0$.

Formally, when upstream firms operate under capacity constraints, each of the M firms sets x_i^k to maximize $\pi_i^{m,k} = w(X^k)x_i^k - C_i^m(x_i^k)$. Note that $C_i^m(x_i^k) > 0$ for $l^k \geq 1$, i.e.,



(a) $k = 0$

(b) $k_1 \to \infty$, $k_2 \in (0, \infty)$, $k_3 = 0$

Figure 1: Upstream equilibrium quantities

capacity constraints matter as soon as one of the downstream firms is licensed (this assumes that the additional input demand that follows from an increase in downstream firms' productive efficiency is divided symmetrically between the upstream firms). In this case, $x_i^k > x_{pre}$ so that $C_i^m(x_i^k) = k(x_i^k - x_{pre})^2/2 > 0$. On the aggregate, this implies that industry output is given by (details are provided in the Appendix)

$$X^k = X_{pre} + \Phi(l^k\theta, k) \tag{5}$$

with $\Phi(l^k\theta, k) = \frac{Ml^k\theta}{(M+1)(N+1)+Nk}$. Expression (5) makes clear that aggregate industry output can be decomposed in its pre-innovation component X_{pre} and a strictly positive component $\Phi(l^k\theta, k)$. For a given number of licensed downstream firms, $\Phi(l^k\theta, k)$ has the following properties:

$$\text{i) } \frac{\partial\Phi(l^k\theta, k)}{\partial k} < 0, \text{ ii) } \frac{\partial\Phi(l^k\theta, k)}{\partial\theta} > 0 \text{ and iii) } \frac{\partial^2\Phi(l^k\theta, k)}{\partial\theta\partial k} < 0 \ \forall\theta > 0. \tag{6}$$

In words, for a given size of the innovation, capacity constraints reduce the aggregate supply of the input factor. Moreover, as already noted in Lemma 1, the innovation results in an expansion of aggregate industry output. This expansion effect, however, strictly decreases in k. In the case of strict capacity constraints, i.e., $k \to \infty$, $\Phi(l^k\theta, k) \to 0$ and $X^k \to X_{pre}$.

As a corollary, capacity constraints lead to an increase in the input and final good price, which, intuitively, is the most pronounced for $k \to \infty$.[10]

Lemma 2 *Assume that at least one downstream firm is licensed so that $C < Nc$. Then, $\forall k \in [0, \infty)$, $X^k \geq X_{pre}$, $p^k \geq p_{pre}$ and $w^k > w_{pre}$. Moreover, $\partial X^k/\partial k < 0$, $\partial p^k/\partial k > 0$ and $\partial w^k/\partial k > 0$.*

[10]The derived demand can be rewritten as $w(X^k) = a - c + \theta l^k/N - X^k(N+1)/N$. It is immediate that θ leads to a demand independent upward shift of $w(X^k)$. The upward shift is balanced by the innovation's expansion effect, i.e., $\partial X^k/\partial\theta \geq 0$. The expansion effect decreases in k, which, in turn, increases the input price.

In the remainder of this section, we solve the second and the first stage of the licensing game for either a per-unit royalty or a fixed fee policy. Then, in Section 3, we derive the optimal licensing contract.

2.2 The per-unit royalty licensing game

Assume first that the innovator transfers the innovation via a per-unit royalty contract. That is, by means of a fee per unit of output produced with the innovation, denoted by r^k. Under such a contract, the innovator's licensing revenues are given by $\pi^{P,r,k} = r^k l^k q_i^{n,l,k}(c_i^l)$ with $c_i^l = c-\theta+r^k$ (that is, a licensed firm's marginal costs decrease in the innovation but increase in the royalty rate). In terms of the timing, the innovator first announces the royalty rate at which a licensing contract may be obtained. Downstream firms then independently and simultaneously decide whether or not to adopt the technology.

To derive the optimal per-unit royalty rate, note first that the maximum r^k a firm is willing to pay for the innovation is θ. Otherwise, the cost of obtaining the innovation (per-unit royalty rate) outweighs its benefit (reduced marginal production costs) and the technology transfer results in an increase of marginal production costs. As a corollary, following the announcement of r^k, with $r^k \leq \theta$, all downstream firms are better off obtaining a licensing contract and the innovator's licensing income is given by $\pi^{P,r,k} = r^k X^k$. For any non-drastic innovation, $\pi^{P,r,k}$ strictly increases in r^k and, as a result, the innovator optimally sets $r^k = \theta$ (details are given in the Appendix).

Lemma 3 *Under the optimal per-unit royalty licensing contract, the innovator licenses non-drastic innovations to all downstream firms and sets the royalty rate equal to the size of the innovation, i.e.,* $l^k = N$ *and* $r^k = \theta$.

At this point, two things are worth noting. First, a per-unit royalty is a variable part in the optimization problem of a licensee and adds to a licensed firm's marginal production costs. In fact, at the licensing equilibrium, the innovator equates royalty rate and cost-reduction. Downstream firms are consequently indifferent between accepting and rejecting a proposed licensing contract and continue producing as they did prior to the transfer of the technology, i.e., at their pre-licensing marginal costs c. As a corollary, with a per-unit royalty contract, the innovation does not result in an expansion of aggregate industry output (*downward distortion* of production levels). This brings with it that the innovator's per-unit royalty licensing strategy and income are not affected by the introduction of capacity constraints. That is, $r^k = \theta$, $l^k = N$ and $\pi^{P,r,k} = \theta X_{pre}\ \forall k \geq 0$.

Lemma 4 *Under a per-unit royalty licensing policy, capacity constraints do not alter the innovator's optimal licensing strategy or income.*

Second, as it will become clear in the following section, under a fixed fee policy the downward distortion of production levels is absent. The technology transfer features an expansion of industry output and by this leads to increased profits of upstream and licensed downstream firms. This is the intuition behind why an outside innovator generally maximizes licensing profits by offering fixed fee instead of per-unit royalty contracts.

2.3 The fixed fee licensing game

Assume now that the innovator opts for a fixed fee contract. Under this licensing policy, the innovator realizes a licensing income of $\pi^{P,f,k} = f^k(l^k)l^k$ where $f^k(l^k)$ denotes the upfront fee that is paid by a licensee. The innovator first announces the fixed fee at which a licensing contract may be obtained. Downstream firms then independently and simultaneously decide whether or not to adopt the technology.

To derive the optimal fixed fee, note first that downstream firms accept any proposed contract as long as the demanded fee, $f^k(l^k)$, does not exceed their willingness to pay for the innovation, $w^k(l^k) = \pi_i^{n,l,k}(l^k) - \pi_i^{n,nl,k}(l^k - 1)$. At the optimum, the innovator sets $f^k(l^k) = w^k(l^k)$ and extracts the entire willingness to pay of the downstream firms.[11] Given downstream firms' profits, the fixed fee can be derived as

$$f^k(l^k) = \theta^2\Big(1 - \frac{\lambda_k}{N}\Big)\Big[\frac{2MA}{(M+1)(N+1)\theta} - \frac{\lambda_k}{N}(2l^k - 1) + 1\Big]. \tag{7}$$

Here, $A = a - c$ and $\lambda_k = \frac{N(M+1)+Nk+1}{(M+1)(N+1)+Nk}$. Notice that $\frac{\partial\lambda_k}{\partial k} > 0$ with $\lambda_0 = \frac{M(N+1)+1}{(M+1)(N+1)}$ and $\lambda_k \to 1$ for $k \to \infty$.[12]

From (7) two things stand out. First, for a given l^k, the inverse demand function for licensing contracts, $f^k(l^k)$, strictly decreases in λ_k. Stricter capacity constraints, hence, reduce the amount a downstream firm is willing to invest in the technology. Second, $f^k(l^k) > 0\ \forall k \in [0, \infty)$ requires $N > 1$. This is why we assume $N \geq 2$ in the following.[13]

In order to determine the optimal fixed fee, the innovator sets l^k to maximize their licensing income. The optimal l^k then defines the licensing income maximizing fee that the innovator announces at the beginning of the game. That is, the innovator solves[14]

$$\max_{l^k} \pi_l^{P,f,k} = f^k(l^k)l^k. \tag{8}$$

[11]More generally, when l^k firms are licensed no licensee has an incentive to deviate (from acceptance to refusal) if and only if $f^k(l^k) \leq w^k(l^k)$. For a non-licensee the condition is given by $f^k(l^k) \geq w^k(l^k + 1)$. Consequently, l^k is optimally supported by $f^k(l^k) = w^k(l^k)$ for $w^k(l^k) \geq w^k(l^k + 1)$. In the given setting this is all the time satisfied.

[12]Taken together, $\lambda_k\theta l^k/N$ measures the indirect effect of the innovation on the price cost margin (PCM) of a downstream firm. That is to say, it captures the effect of the innovation on a downstream firm's PCM via the innovation's effect on aggregate industry output (and by this via the innovation's effect on the input and the final good price). It omits any direct effects of the innovation (i.e., the increase of a licensee's PCM via the cost-reduction, θ). As mentioned previously, under the optimal per-unit royalty contract, the innovation does not alter the industry structure (i.e., does not lead to changes in aggregate output, the input or the final good price) so that $\lambda_k(\theta - r^k)l^k/N = 0$ (under a per-unit royalty contract the net cost reduction is $\theta - r^k$, under a fixed fee contract it is θ).

[13]For instance, $f^M(l^M) = 0$ for $N = 1$ and $k \to \infty$ (the increase in the input price exactly offsets the benefit from the innovation). It is then trivial to show that per-unit royalty contracts yield larger licensing revenues than fixed fee contracts.

[14]For simplicity, we treat the number of licensing contracts as a continuous variable.

Next to an interior solution at $l^{*,k}$ (a strict subset of the downstream market is licensed; licensed and unlicensed firms realize strictly positive market revenues), the optimization problem has two other equilibria at the boundary. One at $l^k = N$ (the entire industry is licensed) and one at $l^k = L^k$ (only licensed firms realize strictly positive market revenues). Lemma 5 summarizes the outcome of the fixed fee licensing game.

Lemma 5 *Under the optimal fixed fee contract, the innovator licenses non-drastic innovations to l^k downstream firms with*

$$l^k = \begin{cases} N & \theta \leq \underline{\theta}^k, \\ \frac{1}{4\lambda_k}\left[\frac{2MNA}{(M+1)(N+1)\theta} + N + \lambda_k\right] & \theta \in (\underline{\theta}^k, \bar{\theta}^k), \\ L^k & \theta \in [\bar{\theta}^k, \bar{\bar{\theta}}^k), \end{cases} \tag{9}$$

where $L^k = \frac{MNA}{(N+1)(M+1)\lambda_k\theta}$, $\underline{\theta}^k = \frac{2MNA}{(M+1)(N+1)(4N\lambda_k - N - \lambda_k)}$, $\bar{\theta}^k = \frac{2MNA}{(M+1)(N+1)(N+\lambda_k)}$, *and* $\bar{\bar{\theta}}^k = \frac{MNA}{(M+1)(N+1)\lambda_k}$. *The optimal fixed fee is given by (7) and (9).*

Notice that the optimal l^k decreases in the strength of capacity constraints. Capacity constraints thus reduce the diffusion of the innovation under a fixed fee policy. Similarly, final good producers' willingness to pay for process technology decreases in k. Taken together it follows that the innovator's fixed fee licensing revenues decrease in the scarcity of the input factor. These results stands in clear contrast to the ones obtained for a per-unit royalty policy ($r^k = \theta$, $l^k = N$ and $\pi^{P,r,k} = \theta X_{pre}\ \forall k \geq 0$).

Lemma 6 *Under a fixed-fee contract, capacity constraints impact the innovator's optimal licensing strategy and income. In particular, stricter capacity constraints result in a lower fixed fee, a lower uptake of licensing contracts and decreased licensing revenues.*

3 The optimal licensing contract

Having derived the optimal fixed fee and per-unit royalty licensing contract, we now compare the innovator's licensing income across policies and identify the conditions under which the innovator prefers one policy over the other. Here, our focus is again on highlighting the impact of capacity constraints on the innovator's optimal licensing strategy.

To do so, we start with a scenario in which capacity constraints are strict ($k \rightarrow \infty$). As mentioned previously, this corresponds, for instance, to a scenario in which upstream firms operate at their capacity level prior to the technology transfer and are reluctant to invest in their capacities (prohibitively high costs, uncertainty of demand conditions) or face a significant time lag between the investment and the capacity expansion. We then study how our results are affected by a softening of capacity constraints ($k \in (0, \infty)$). In each section, we begin with a simple duopoly example and then extend the analysis to an oligopolistic market environment.

3.1 Strict capacity constraints

A duopoly example

To illustrate the main result of our paper, we initially restrict our attention to a vertical Cournot duopoly ($M = N = 2$). Under strict capacity constraints, both upstream firms cannot produce more than in the pre-licensing equilibrium.

Let us first consider the case of a per-unit royalty contract. From Lemma 3 it directly follows that $r^k = \theta$ and $l^k = 2$. The associated licensing revenues are $\pi^{P,r,\infty} = 2x_{pre}\theta = 4A\theta/9$. In fact, as we argue previously, this result applies irrespective of the value of k.

Under a fixed fee policy, the innovator transfers the innovation either to both or to a single downstream firm. From Lemma 5 it follows that the innovator optimally licenses both downstream firms whenever the innovation is sufficiently small. For larger cost-reductions, a single firm is licensed. The optimal licensing strategy and the associated licensing revenues for respectively full (10) and no (11) capacity constraints are consequently

$$\pi_l^{P,f,\infty} = \begin{cases} \frac{18\theta^2}{81}(\frac{2A}{\theta} - \frac{9}{4}) & \theta \le \frac{8A}{27}, \\ \frac{9\theta^2}{81}(\frac{2A}{\theta} + \frac{9}{4}) & \theta \in (\frac{8A}{27}, \frac{4A}{9}), \end{cases} \tag{10}$$

and

$$\pi_l^{P,f,0} = \begin{cases} \frac{22\theta^2}{81}(\frac{2A}{\theta} - \frac{3}{4}) & \theta \le \frac{8A}{17}, \\ \frac{11\theta^2}{81}(\frac{2A}{\theta} + \frac{11}{4}) & \theta \in (\frac{8A}{17}, \frac{4A}{7}). \end{cases} \tag{11}$$

Comparing $\pi^{P,r,0}$ to $\pi_l^{P,f,0}$, one first observes that in the absence of capacity constraints the innovator maximizes licensing revenues by offering a fixed fee contract. This result is due to the previously mentioned downward distortion of production levels under a per-unit royalty contract. However, a comparison of $\pi^{P,r,\infty}$ and $\pi_l^{P,f,\infty}$ shows that the introduction of capacity constraints reverses this result. As such, under strict capacity constraints, the innovator maximizes licensing revenues by offering a per-unit royalty contract. To summarize, $\pi^{P,r,0} < \pi_l^{P,f,0}$, whereas $\pi^{P,r,\infty} > \pi_l^{P,f,\infty}$.

Our result can be rationalized by the presence of a *capacity effect*, which reduces the innovator's fixed fee licensing income for a given size of the innovation. To see this, note that under a fixed fee contract the technology reduces the marginal production costs of any licensed downstream firm. This leads to an expansion of downstream production levels and by this to an increased demand for the input factor (one-to-one production technology). However, with strict capacity constraints, upstream firms cannot accommodate this higher level of downstream demand. As a corollary, capacity constraints result in an increase of the input price. This increase reduces downstream firms' willingness to pay for the innovation and by this lowers the innovator's fixed fee licensing income. Intuitively, the capacity effect increases in the size of the innovation (and more generally, for $k \in [0, \infty)$, in the strictness of the capacity constraints). Under a per-unit royalty contract, this effect is absent as the innovation does not lead to an expansion of downstream production levels.

In this context, we want to point to another result which illustrates the working of the capacity effect. Note that under a fixed fee contract, the innovator prefers to license both firms if and only if the innovation is small. Here, one can observe that the critical threshold is lower in the presence of (strict) capacity constraints ($\theta \leq 8A/27$ versus $\theta \leq 8A/17$). For a given size of the innovation, both firms producing with the superior technology implies a larger level of downstream derived demand and by this a larger capacity effect. It follows that under capacity constraints licensing to both downstream firms is optimal only for sufficiently small innovations. Capacity constraints, hence, make a full diffusion of the innovation less likely, and, in the given setting, may explain why the innovator may prefer to license a single downstream firm exclusively.

Oligopolistic industries

We now discuss how the previously derived results may be generalized for an industry with an oligopolistic upstream and downstream sector. While in a vertical Cournot duopoly it is all the time optimal to offer a per-unit royalty contract as $k \to \infty$, this result does not necessarily apply in a vertical Cournot oligopoly. Instead, our findings show that the superiority of a per-unit royalty over a fixed fee contract in an oligopolistic market environment depends on the presence of a sufficiently strong capacity effect.

From the characterization of the optimal per-unit royalty and fixed fee licensing contract it is clear that under a per-unit royalty contract $l^k = N$ irrespective of the size of the innovation. In contrast, for a fixed fee policy we have three subcases ($l^k = L^k$, $l^k = l^{*,k}$, $l^k = N$) that depend on θ. Consequently, in order to derive the optimal licensing strategy, it is necessary to compare the innovator's licensing revenues across policies for all three subcases (we provide analytical details in the Appendix).

Large innovations (i.e., $\theta \in [\bar{\theta}^k, \bar{\bar{\theta}}^k)$ and $l^k = L^k$). For sufficiently large values of the innovation, only licensed firms realize strictly positive market revenues. As mentioned previously, we restrict the attention to cases where at least two licensed firms make positive profits (i.e., we consider non-drastic innovations).

Lemma 7 *With a fully constrained upstream sector, large innovations are optimally licensed via per-unit royalty contracts. That is, $\pi^{P,r,\infty} \geq \pi_L^{P,f,\infty}$ for all $\theta \in [\bar{\theta}^\infty, \bar{\bar{\theta}}^\infty)$. Note that in the absence of capacity constraints $\pi^{P,r,0} \geq \pi_L^{P,f,0}$ if and only if $M = 1, N = 2$.*

Intuitively, large innovations feature the largest values of non-drastic process innovations and by this the most pronounced capacity effects. For large innovations, capacity constraints therefore cause the strongest increase in the input price and by this the largest reduction in the innovator's fixed fee licensing income. In the given framework, large innovations are consequently always licensed via per-unit royalty contracts.

Intermediate innovations (i.e., $\theta \in (\underline{\theta}^k, \bar{\theta}^k)$ and $l^k = l^{*,k}$). For intermediate values of the innovation licensed and unlicensed firms realize strictly positive market revenues.

Lemma 8 *With a fully constrained upstream sector, intermediate innovation are optimally licensed via per-unit royalty contracts for either a sufficiently concentrated downstream sector or* *a sufficiently large cost-reduction. That is,* $\pi^{P,r,\infty} \geq \pi_{l^*}^{P,f,\infty}$ *if and only if either* $N \leq 3$ *and* $\theta \in (\underline{\theta}^{\infty}, \bar{\theta}^{\infty})$ *or* $N > 3$ *and* $\theta \in [\theta_{2,-}^{\infty}, \bar{\theta}^{\infty})$ *(where* $\theta_{2,-}^{\infty} > \underline{\theta}^{\infty}$*). Note that in the absence of capacity constraints* $\pi^{P,r,0} \geq \pi_{l^*}^{P,f,0}$ *if and only if* $M = 1, N = 2$.

Lemma 8 implies that for a sufficiently concentrated downstream market it is all the time optimal to transfer an intermediate innovation by means of a per-unit royalty contract. For a more competitive downstream market, the innovation has to be of a certain size in order to ensure a minimal capacity effect. To see the intuition behind this result, notice that for $\theta \in (\underline{\theta}^{\infty}, \bar{\theta}^{\infty})$ the innovation, and by this also the capacity effect and the reduction in the innovator's fixed fee licensing income, are of an intermediate size. This implies that the advantage of a per-unit royalty over a fixed fee contract in terms of licensing income is less pronounced for intermediate, as compared to large, innovations. Moreover, as N increases, $\pi^{P,r,\infty}$ and $\pi_{l^*}^{P,f,\infty}$ both increase, however, $\pi_{l^*}^{P,f,\infty}$ does so at a faster rate. Consequently, a sufficiently large N may compensate for the disadvantage of a fixed fee contract in terms of licensing revenues. It follows that for a sufficiently competitive downstream market, a lower bound on θ is required to ensure the optimality of a per-unit royalty over a fixed fee contract.

Minor innovations (i.e., $\theta \in (0, \underline{\theta}^{k}]$ *and* $l^k = N$*).* For sufficiently small innovations, the entire downstream industry is licensed.

Lemma 9 *With a fully constrained upstream sector, minor innovations are optimally licensed via per-unit royalty contracts for either a downstream duopoly or a downstream triopoly and a sufficiently large cost-reduction. That is,* $\pi^{P,r,\infty} \geq \pi_{N}^{P,f,\infty}$ *if and only if* $N \leq 3$ *and* $\theta \in [\theta_1^{\infty}, \underline{\theta}^{\infty}]$. *Note that in the absence of capacity constraints there are no integer values of* M, N *such that* $\pi^{P,r,0} \geq \pi_{N}^{P,f,0}$.

Similar to Lemma 8, Lemma 9 states that a per-unit royalty contract is optimal for sufficiently concentrated downstream industries. What distinguishes the result in Lemma 9 from the one in Lemma 8 is the fact that for any $N > 3$ per-unit royalty contracts never maximize licensing revenues for minor innovations. How may this result be rationalized? For $\theta \in (0, \underline{\theta}^{\infty}]$, the innovation is small. Consequently, the capacity effect, which may render a per-unit royalty contract the optimal licensing policy, is equally small. Also, as N increases, $\pi_{N}^{P,f,\infty}$ increases at a very fast rate ($\pi_{N}^{P,f,\infty} = f^{\infty}(l^{\infty})l^{\infty}$ with $l^{\infty} = N$). Taken together, this implies that per-unit royalty contracts yield only marginally larger licensing revenues than fixed fee contracts. Further, this difference in profitability is easily compensated by a small increase in N. It follows that for $N > 3$ the innovator optimally licenses minor innovations via fixed fee contracts.

Proposition 1 *Assume that an outside innovator licenses a process innovation to the downstream sector of a vertical Cournot oligopoly with strict capacity constraints in the upstream sector. In this setting, innovations that feature sufficiently large capacity effects are optimally licensed via per-unit royalty contracts. Conversely, innovations that feature only minor capacity effects are optimally licensed via fixed fee contracts. For strict capacity constraints, a sufficiently strong capacity effect translates into a lower bound on the size of the innovation and/or an upper bound on the size of the downstream sector.*

3.2 Soft capacity constraints

The previous results are derived under the assumption of strict capacity constraints ($k \to \infty$). Although this scenario serves well to illustrate the main mechanisms at work, it may bee seen as restrictive. That is why in the present section we address soft capacity constraints ($k \in (0, \infty)$). As in the previous section, we start with a simple duopoly example and then extend our results to the oligopolistic case.

A duopoly example

It has been stated repeatedly throughout this paper that the innovator's royalty licensing strategy and income are unaffected by the introduction of capacity constraints. Hence, $\forall k \geq 0\ r^k = \theta$, $l^k = 2$ and $\pi^{P,r,k} = 2x_{pre}\theta = 4A\theta/9$.

Under a fixed fee policy, sufficiently small innovations are licensed to both downstream firms and large innovations to a single downstream firm. The innovator's associated licensing revenues are given by

$$\pi_l^{P,f,k} = \begin{cases} 2\theta^2(1 - \frac{\lambda_k}{2})(\frac{4A}{9\theta} + 1 - \frac{3\lambda_k}{2}) & \theta \leq \frac{8A}{9(5\lambda_k - 2)}, \\ \theta^2(1 - \frac{\lambda_k}{2})(\frac{4A}{9\theta} + 1 - \frac{\lambda_k}{2}) & \theta \in \big(\frac{8A}{9(5\lambda_k - 2)}, \frac{4A}{9\lambda_k}\big) \end{cases} \tag{12}$$

with $\lambda_k = \frac{7+2k}{9+2k}$ at $M = N = 2$.

Assume that $\theta \leq 8A/9(5\lambda_k - 2)$ so that both downstream firms adopt the process technology, irrespective of the type of licensing policy. Comparing the innovator's licensing revenues across policies it immediately follows that $\pi^{P,r,k} \geq \pi_2^{P,f,k}$ for any $\theta \geq 8A(1-\lambda_k)/9(2-\lambda_k)(3\lambda_k - 2)$ and $k \geq (\sqrt{17} - 4)/2 \approx 0.06155$. This result illustrates nicely that in the absence of capacity constraints ($k = 0$) fixed fee always dominate per-unit royalty contracts. As soon as the expansion of production levels above the status-quo threshold x_{pre} becomes marginally costly, this result may be reversed in favor of per-unit royalty contracts for sufficiently important innovations.

Oligopolistic industries

From the analysis so far it is clear that innovations that feature sufficiently strong capacity effects are optimally licensed via per-unit royalty contracts. In the previous section, we showed that for strict capacity constraints a sufficiently strong capacity effect translates into a lower bound on the size of the innovation (potentially depending on the size of the downstream sector). However, it is intuitive that generally, the size of the capacity effect not only increases in θ but also in k. The larger k, the less the upstream sector is able to accommodate the increased demand for the input factor that follows the adoption of the process innovation by the downstream firms. As a corollary, for a fixed fee contract, an increase in k increases the input price and by this lowers a downstream firm's willingness to pay for the innovation as well as the number of downstream firms that accept a licensing contract. All in all, this implies that $\partial\pi_l^{P,f,k}/\partial k < 0$, whereas $\partial\pi^{P,r,k}/\partial k = 0$ (this is formally shown in the Appendix). It follows that an increase in k makes it more likely that a per-unit royalty contract maximizes the innovator's licensing revenues.

Based on this discussion, we now show how the previously derived results may be generalized for any $k \in (0, \infty)$. Assume that $M, N \geq 2$ and $\pi^{P,r,k} \geq \pi_l^{P,f,k}$ for $k \to \infty$.[15] Then, there is a unique value $k^* \in (0, \infty)$ for which the innovator is indifferent between a per-unit royalty and a fixed fee contract. For sufficiently strict capacity constraints, i.e., for $k \in (k^*, \infty)$, the innovator maximizes licensing revenues by means of a per-unit royalty contract; otherwise, fixed fee contracts are optimal.

Proposition 2 *Assume that an outside innovator licenses a process innovation to the downstream sector of a vertical Cournot oligopoly with a capacity constrained upstream sector. Assume further that* $M, N \geq 2$ *and* $\pi^{P,r,\infty} \geq \pi_l^{P,f,\infty}$. *Then, there is a unique value* $k^* \in (0, \infty)$ *such that* $\pi^{P,r,k^*} = \pi_l^{P,f,k^*}$. *For* $k \in (0, k^*)$ $\pi_l^{P,f,k} > \pi^{P,r,k}$, *for* $k \in [k^*, \infty)$ $\pi^{P,r,k} \geq \pi_l^{P,f,k}$.

Proof This result directly follows from the fact that i) $\pi^{P,r,k} \geq \pi_l^{P,f,k}$ for $k \to \infty$, ii) $\pi_l^{P,f,0} > \pi^{P,r,0}$ for $M, N \geq 2$, iii) $\frac{\partial \pi_l^{P,f,k}}{\partial k} < 0$, iv) $\frac{\partial \pi^{P,r,k}}{\partial k} = 0$. □

At this point, it is instructive to comment on how our results depend on the choice of capacity level, i.e., on $\bar{x} = x_{pre}$.

To recall, per-unit royalty payments raise licensed firms' marginal production costs to their pre-innovation level. Under a per-unit royalty contract, the innovation thus does not result in a net-efficiency gain on the downstream market and therefore does not feature any expansion of production levels. Under a fixed-fee contract, this effect is absent which explains why this type of contract typically yields a higher licensing income for independent technology providers.

It is clear that in order to reverse this result in favor of per-unit royalty contracts it is necessary to penalize the output expansion under fixed fee contracts in a way that reduces the innovator's licensing income (i.e., in a way that lowers downstream firms' willingness to pay for cost-reducing process technology).

In the given framework, i.e., for $\bar{x} = x_{pre}$, any expansion beyond the benchmark production level (no technology transfer) involves additional production costs on the upstream market and, by this, reduces final good producers' willingness to invest in process technology and hence the innovator's fixed fee revenues. It is then immediate that setting $\bar{x} > x_{pre}$ makes it less likely that per-unit royalty contracts are superior (for $\bar{x} = x_i^0$ upstream firms are unconstrained and can expand their production at no additional cost). For $\bar{x} < x_{pre}$, capacity constraints not only reduce the innovator's fixed fee revenues but also their per-unit royalty income. Nevertheless, per-unit royalties continue to have a certain advantage over fixed-fee contracts due to their lower output expansion.[16]

[15] As explain previously, $N > 1$ ensures that $f^k(l^k) > 0 \; \forall k$; $M \geq 2$ rules out cases in which $\pi^{P,r,0} \geq \pi_l^{P,f,0}$ (i.e., in which per-unit royalty contracts are optimal irrespective of the value of k). If, for a given size of the innovation, $\pi^{P,r,k} < \pi_l^{P,f,k}$ for $k \to \infty$, per-unit royalty contracts are never optimal.

[16] Take the case of a vertical Cournot oligopoly ($M = N = 2$) and assume that $\bar{x} = 0$ (upstream firms face convex costs also in the absence of a technology transfer to the downstream market). In that case $r^k = \theta$, $l^k = 2$ and $\pi^{P,r,k} = 4A\theta/(9+2k)$ under the optimal per-unit royalty contract. Assume that $\theta \leq 8A/(9+2k)(5\lambda_k - 2)$ so that $l^k = 2$ is also optimal under a fixed fee contract. In that case, $\pi^{P,f,k} = 2\theta^2(1-\lambda_k/2)(4A/(9+2k)+1-3\lambda_k/2)$ where as before $\lambda_k = (7+2k)/(9+2k)$. It is easy to show that $\pi^{P,r,k} \geq \pi_2^{P,f,k}$ for any $\theta \geq 8A(1-\lambda_k)/(9+2k)(2-\lambda_k)(3\lambda_k - 2)$ and $k \geq (\sqrt{17}-4)/2 \approx 0.06155$. The critical threshold on θ is thus lower than under $\bar{x} = x_{pre}$.

4 Welfare outcomes and the diffusion of process technology

In the previous sections, we argue that an increase in the scarcity of an input factor makes it more likely that an outside innovator chooses a per-unit royalty over a fixed fee contract when licensing process technology. One may wonder how this conclusion extends to the social optimality of per-unit royalty contracts, as measured by aggregate welfare or the diffusion of the innovation.

4.1 Welfare outcomes

We begin by briefly discussing some welfare aspects of the privately optimal licensing contract. First, we compare consumer and producer surplus across licensing policies. In a second step, we then contrast the socially optimal licensing policy with the one that is chosen by the innovator.

Lemma 10 *Denote by CS_s^k, $\Pi_s^{m,k}$ and $\Pi_s^{n,k}$ with $s \in \{fee,\ royalty,\ pre\}$ consumer and producer surplus under eiher a fixed fee contract, a per-unit royalty contract or in the pre-licensing equilibrium. We observe that consumers weakly prefer a fixed fee to a per-unit royalty contract ($CS_{fee}^k \geq CS_{royalty}^k = CS_{pre}$), upstream firms strictly prefer a fixed fee to a per-unit royalty contract ($\Pi_{fee}^{m,k} > \Pi_{royalty}^{m,k} = \Pi_{pre}^m$) and downstream firms strictly prefer a per-unit royalty to a fixed fee contract ($\Pi_{pre}^n = \Pi_{royalty}^{n,k} > \Pi_{fee}^{n,l,k} > \Pi_{fee}^{n,nl,k}$).*

Lemma 10 illustrates that upstream and downstream firms generally have conflicting interests regarding the optimal, surplus maximizing, licensing contract. While upstream firms prefer a fixed fee contract, downstream firms are better off under a per-unit royalty contract. What is more, the innovator's private incentives are aligned with those of the firms in the licensed industry, whenever a royalty contract is privately optimal.

In the following, we shed some light on the welfare implications of the privately optimal licensing policy (details are given in the Appendix). To do so, we define aggregate welfare, $W_{royalty}^k$ and W_{fee}^k, as the sum of consumer surplus, producer surplus and licensing revenues.

Proposition 3 *Depending on the size of the innovation, either a per-unit royalty or a fixed fee licensing contract maximizes aggregate welfare. For minor or large innovations, i.e., $\theta \in (0, \underline{\theta}^k]$ or $\theta \in [\bar{\theta}^k, \bar{\bar{\theta}}^k]$, $W_{royalty}^k \leq W_{fee}^k$. For intermediate innovations, i.e., $\theta \in (\underline{\theta}^k, \bar{\theta}^k)$, $W_{royalty}^k \geq W_{fee}^k$ (for large values of k) or $W_{royalty}^k < W_{fee}^k$ (for small values of k).*

From Proposition 3 two main observations stand out. First, in contrast to the standard licensing literature, per-unit royalty contracts may yield a strictly larger aggregate welfare than fixed fee contracts. Second, there is scope for conflict between the privately and the socially optimal licensing policy; either because the innovator chooses a per-unit royalty contract for minor, intermediate (together with small values of k) or large innovations or because they license intermediate innovations via fixed fee contracts (together with sufficiently large values of k).

4.2 The diffusion of process technology

Clearly, for a given licensing policy the diffusion of the innovation (as measured by l^k) weakly decreases in k. As such, $\partial l^k/\partial k = 0$ for a per-unit royalty contract and $\partial l^k/\partial k \leq 0$ for a fixed fee contract. However, as the previous sections demonstrate, changes in k may equally imply changes in the privately optimal licensing policy. In particular, with stricter capacity constraints the innovator is more likely to opt for a per-unit royalty contract. Under the latter type of contract, the innovation is fully diffused (i.e., licensed to the entire industry). A priori, the question of how capacity constraints influence the diffusion of process technology is thus ambiguous.

We illustrate this point by means of a duopoly example ($M = N = 2$, $k \geq 0$). From the previous analysis it is clear that for $\theta \leq 8A/17$ the innovator optimally chooses a fixed fee contract and licenses the innovation to both downstream firms, whenever $k = 0$. For $k \geq (\sqrt{17} - 4)/2$, a per-unit royalty contract may become optimal. Thus, for $\theta \leq 8A/17$, $l^k = 2\ \forall k$ and an increase in k does not imply a change in the diffusion of the innovation (despite a potential change in licensing policy). In contrast, for $\theta > 8A/17$ the innovation is licensed to a single downstream firm via a fixed fee contract, whenever $k = 0$. Again, for $k \geq (\sqrt{17} - 4)/2$ a per-unit royalty contract may become optimal. Taken together, this implies that an increase in the scarcity of the input factor may increase the diffusion of sufficiently large innovations.

More generally, whenever a per-unit royalty is the optimal *status-quo* licensing policy an increase in k does not entail a change in the diffusion of the innovation (we showed previously that the optimality of royalty over fixed fee contracts increases in k). In contrast, whenever a fixed fee is the optimal *status-quo* policy, a sufficiently strong increase in k (meaning, sufficiently strong to induce a change in the optimal licensing policy from a fixed fee to a per-unit royalty) strictly increases l^k for sufficiently large innovations, i.e., for $\theta > \underline{\theta}^k$ (for $\theta \leq \underline{\theta}^k$ $l^k = N$ irrespective of the type of licensing contract).

5 Extension: two-part tariffs

In the previous section, we identified the scarcity of an essential production factor as a potential explanation for the use of per-unit royalty rates in licensing contracts. The intuition that underlies our main results also carries over to two-part tariff licensing contracts, i.e., to the combination of per-unit royalty and fixed fee payments.

Take, for example, the case of a vertical Cournot duopoly with general capacity constraints and assume that the innovation is licensed to both downstream firms ($M = N = 2$, $k \in [0, \infty)$ and $l^k = 2$).

For $l^k = 2$ the innovator sets r^k in order to maximize

$$\pi_2^{P,t,k} = r^k 2q_i^{n,l,k} + 2f^k(2). \tag{13}$$

It follows that the optimal royalty rate is given by

$$r^k = \frac{1}{\lambda_k(4-3\lambda_k)}\Big\{\theta\big[3\lambda_k(2-\lambda_k)-2\big] - \frac{4A}{9}(1-\lambda_k)\Big\}.^{17} \tag{14}$$

From (14) it is clear that under strict capacity constraints $r^\infty = \theta$, i.e., for $k \to \infty$, the optimal two-part tariff reduces to a pure per-unit royalty contract. For $k \in (0,\infty)$, $r^k < \theta$ and the innovator licenses the process innovation via either a pure fixed fee contract or a two-part tariff. Here, the intuition developed in the previous sections applies. Sufficiently small innovations are optimally licensed via pure fixed fee contracts. In contrast, for sufficiently large innovations, the optimal licensing contract includes a strictly positive royalty rate. More precisely, for $\theta \leq \frac{4A}{9}\big[\frac{1-\lambda_k}{3\lambda_k(2-\lambda_k)-2}\big]$ $r^k = 0$ and the innovator offers a pure fixed fee contract (we exclude the possibility of negative royalty rates). For $\theta > \frac{4A}{9}\big[\frac{1-\lambda_k}{3\lambda_k(2-\lambda_k)-2}\big]$, the royalty component is strictly positive and given by (14). Notice that an increase in k implies that the optimal contract puts more weight on the royalty component. This follows from two observations. First, the critical threshold on θ, i.e., $\frac{4A}{9}\big[\frac{1-\lambda_k}{3\lambda_k(2-\lambda_k)-2}\big]$, decreases in k. Higher values of k thus make it more likely that the optimal contract includes a positive royalty rate. Second, the optimal r^k increases in k so that higher values of k are associated with larger royalty rates.

6 Conclusion

Over the past decades, raw materials scarcity has become a topic of growing concern. Contrary to common belief, the underlying reason is not an impending threat of a depletion of natural resources, but rather an imbalance between raw materials demand and supply. In this context, especially emerging technologies place an increasing strain on the supply of various technology relevant minerals and metals. In light of this evidence, this paper aims to shed light on the question of how such a demand-side scarcity of essential production factors influences technology transfer strategies. Here, we focus our attention on one main channel of technology transfer agreements, namely licensing contracts.

As such, we study the licensing strategy of an independent, i.e., non-producing, innovator who licenses a process innovation to the downstream sector of a vertical Cournot oligopoly. Downstream firms source an essential production factor from an oligopolistic upstream sector in which firms operate under capacity constraints. Here, the upstream firms' supply restrictions arise as a corollary of the adoption of the process technology in the downstream sector. The main part of this paper studies and compares fixed fee and per-unit royalty contracts in terms of the innovator's licensing revenues and their welfare properties. In an extension, we also address two-part tariffs.

We show that capacity constraints discriminate between fixed fee and per-unit royalty contracts in terms of the innovator's licensing strategy and income. Most important, under a fixed fee contract the innovation features a capacity effect. As such, the adoption of the process

[17] For $l^k = 1$, the optimal royalty rate is given by $r^k = \big(\frac{\lambda_k - 1}{\lambda_k}\big)\big[\frac{4A}{9(2-\lambda_k)} + \theta\big]$. Note that $r^k \leq 0$ $\forall \lambda_k$ so that the innovator all the time opts for a pure fixed fee contract.

technology by the downstream firms leads to an increased demand for the input factor. Under capacity constraints, however, upstream firms can satisfy this increased demand only imperfectly. As a corollary, the input price increases which in turn reduces the net-benefit of the process technology for a downstream firm. It follows that under a fixed fee policy, capacity constraints reduce a downstream firms' willingness to pay for the innovation as well as the number of downstream firms that accepts a licensing contract. For a per-unit royalty policy, in contrast, such a capacity effect is absent. It is thus intuitive that innovations that feature sufficiently large capacity effects are optimally licensed via per-unit royalty contracts. Here, the size of the capacity effect increases in the size of the innovation and the strength of the capacity constraints.

Our analysis leaves room for further extensions. First, in our eyes, it would be interesting to consider the case in which upstream capacity constraints do not affect all downstream firms equally, for example, because only some downstream firms face capacity constrained suppliers. In this case, does the innovator find it profitable to discriminate between different types of downstream firms by offering different licensing contracts (or types of licensing contracts)? Do downstream firms have an incentive to stay with a constrained supplier as this allows them to secure a more advantageous licensing contract? Similarly, it would be interesting to consider scenarios in which resource scarcity affects firms heterogeneously, for instance, depending on factors such as their ability to manage resource risks or the efficiency of their production processes. Second, in the analysis so far we abstract from capacity investments by the upstream firms. Nevertheless, it would be important to shed light on the question of whether upstream firms have an incentive to invest in a softening of their capacity constraints and how those incentives relate to the innovator's licensing strategy. Related is the following point. Policy initiatives such as recycling, urban mining or substitution aim at addressing (or preventing) mineral and metals scarcity. Our analysis provides a starting point to study the relationship between such initiatives and the development and diffusion of process technology. Finally, we consider innovations that increase the efficiency of downstream firms' production processes and by this lower their marginal production costs. In the context of resource scarcity, it would be important to also study the licensing of technologies that result in a more efficient usage of the input factor and by this reduce downstream firms' resource needs.

7 Appendix

7.1 Market stage analysis

Assume that on the downstream sector $l^k \in [0, N]$ firms are licensed. A typical downstream firm maximizes a profit function of the form $\pi^{n,\alpha,k} = \big(p(Q^k) - w - c_i^{\alpha}\big) q_i^{\alpha,k}$ with respect to its individual level of production, $q_i^{\alpha,k}$. To recall $p(Q^k) = a - Q^k$ denotes the inverse demand function, w the input price, $\alpha \in \{l, nl\}$ the licensing status of a downstream firm (l=*licensed*, nl=*not licensed*) and $k \in [0, \infty)$ refers to the strictness of the upstream firms' capacity constraints. Licensed (unlicensed) downstream firms produce with marginal cost $c_i^l = c - \theta$ ($c_i^{nl} = c$).

The optimization problems of respectively licensed and unlicensed downstream firms therefore are

$$\forall i \in 1,\dots,l^k : \max_{q_i^{l,k}} \pi_i^{n,l,k} = \big[a - \big(q_i^{l,k} + (l^k-1)q_j^{l,k} + (N-l^k)q_i^{nl,k}\big) - w - c_i^l\big]q_i^{l,k},$$
$$\forall i \in l^k+1,\dots,N : \max_{q_i^{nl,k}} \pi_i^{n,nl,k} = \big[a - \big(q_i^{nl,k} + (N-l^k-1)q_j^{nl,k} + l^k q_i^{l,k}\big) - w - c_i^{nl}\big]q_i^{nl,k}. \tag{15}$$

From (15) we derive the best-response function of a typical licensed and unlicensed downstream firm as

$$q_i^{l,k}(q_i^{nl,k}) = \Big(\frac{1}{l^k+1}\Big)\big(a - w - c + \theta - (N-l^k)q_i^{nl,k}\big),$$
$$q_i^{nl,k}(q_i^{l,k}) = \Big(\frac{1}{N-l^k+1}\Big)\big(a - w - c - l^k q_i^{l,k}\big). \tag{16}$$

It follows that, depending on the licensing status α, the production level of a downstream firm is given by

$$q_i^{\alpha,k} = \begin{cases} \big(\frac{1}{N+1}\big)\big(a - w - c + \theta + (N-l^k)\theta\big) & \alpha = l, \\ \big(\frac{1}{N+1}\big)(a - w - c - l^k\theta) & \alpha = nl. \end{cases} \tag{17}$$

Aggregate industry output $Q^k(w)$ is thus

$$Q^k(w) = l^k q_i^{l,k} + (N-l^k)q_i^{nl,k} = \frac{N\big(a - w - \frac{C}{N}\big)}{N+1} \tag{18}$$

with $C = (N-l^k)c + l^k(c-\theta)$.

Re-arrangement of (18) yields the indirect derived demand for the input factor $w(Q^k)$ with

$$w(Q^k) = a - \frac{C}{N} - \Big(\frac{N+1}{N}\Big)Q^k. \tag{19}$$

Given $Q^k = X^k$, upstream firms maximize $\pi_i^{m,k} = w(X^k)x_i^k - C_i^m(x_i^k)$ with respect to x_i^k. Here,

$$C_i^m(x_i^k) = \begin{cases} 0 & x_i^k \le x_{pre}, \\ \frac{k}{2}(x_i^k - x_{pre})^2 & x_i^k > x_{pre}, \end{cases} \tag{20}$$

where $x_{pre} = \frac{N(a-c)}{(M+1)(N+1)}$ denotes the production level of an upstream firm in the pre-licensing equilibrium.

Then,

$$\forall\ i \in 1,\dots,M : \max_{x_{i,}^k} \pi_i^{m,k} = \big[a - \frac{C}{N} - \Big(\frac{N+1}{N}\Big)\big(x_i^k + (M-1)x_{-i}^k\big)\big]x_i^k - \frac{k}{2}(x_i^k - x_{pre})^2 \tag{21}$$

gives the optimization problem of a typical upstream firm.

From the corresponding first-order conditions it is easily derived that

$$x_i^k = \frac{N(a-c)}{(M+1)(N+1)} + \frac{l^k\theta}{(M+1)(N+1)+Nk} \tag{22}$$

so that

$$X^k = \frac{MN(a-c)}{(M+1)(N+1)} + \frac{Ml^k\theta}{(M+1)(N+1)+Nk} = X_{pre} + \Phi(l^k\theta, k). \tag{23}$$

From (17), (19), (23) and $p(Q^k) = a - Q^k$ it follows that downstream profits are

$$\pi_i^{n,\alpha,k} = \begin{cases} \Big(\frac{MA}{(M+1)(N+1)} + \theta(1 - \frac{l^k}{N}\lambda_k)\Big)^2 & \alpha = l, \\ \Big(\frac{MA}{(M+1)(N+1)} - \theta\frac{l^k}{N}\lambda_k\Big)^2 & \alpha = nl, \end{cases} \tag{24}$$

where $\lambda_k = \frac{N(M+1)+Nk+1}{(M+1)(N+1)+Nk}$.

7.2 The per-unit royalty licensing game

At the beginning of the per-unit royalty licensing game, the innovator announces the per-unit royalty rate r^k where $r^k \leq \theta$.[18] For $r^k \leq \theta$ each downstream firm is better off accepting a licensing contract and $l^k = N$.

It follows that the innovator's optimization problem is

$$\max_{r^k} \pi^{P,r,k} = r^k N q_i^{n,l,k}(c_i^l) = r^k N\Big[\frac{MA}{(M+1)(N+1)} + (\theta - r^k)(1-\lambda_k)\Big]. \tag{25}$$

where $r^k \leq \theta$.

Note that $\pi^{P,r,k}$ is strictly increasing in r^k, i.e., $\frac{\partial \pi^{P,r,k}}{\partial r^k} > 0$, if and only if

$$\frac{MA}{(M+1)(N+1)} + \theta(1-\lambda_k) > 2r^k(1-\lambda_k). \tag{26}$$

Clearly, for $\lambda_k = 1$ (26) is satisfied. For $\lambda_k < 1$ the condition reduces to

$$r^k < \frac{1}{2}\Big[\frac{MA}{(M+1)(N+1)(1-\lambda_k)} + \theta\Big]. \tag{27}$$

Given $r^k \leq \theta$ it is sufficient to show that

$$\theta < \frac{1}{2}\Big[\frac{MA}{(M+1)(N+1)(1-\lambda_k)} + \theta\Big] \tag{28}$$

or

$$\theta < \frac{MA}{(M+1)(N+1)(1-\lambda_k)}. \tag{29}$$

This condition is satisfied for any non-drastic innovation, i.e., for any $\theta < \frac{MNA}{(M+1)(N+1)\lambda_k}$.

[18] The upper bound on r^k is easily derived by comparing a downstream firm's profits with and without a licensing contract. As such, $\pi^{n,l,k}(l^k) \geq \pi^{n,nl,k}(l^k - 1)$ if and only if $(\theta - r^k)(1 - l^k\lambda_k/N) + (\theta - r^k)(l^k - 1)\lambda_k/N \geq 0$.

It follows that the privately optimal per-unit royalty licensing contract is characterized by $r^k = \theta$. Together with $l^k = N$ this yields $\pi^{P,r,k} = \theta X_{pre}$.

7.3 The fixed fee licensing game

Under a fixed fee policy, the innovator's optimization problem is given by

$$\max_{l^k} \pi^{P,f,k} = f^k(l^k)l^k \tag{30}$$

with $l^k \leq \min\{L^k, N\}$, $L^k = \frac{MNA}{(M+1)(N+1)\lambda_k\theta} > 1$ and $f^k(l^k)$ given by (7).[19] It follows that $\frac{\partial f^k(l^k)l^k}{\partial l^k} = 0$ at

$$l^{k,*} = \frac{1}{4\lambda_k}\Big[\frac{2MNA}{(M+1)(N+1)} + N + \lambda_k\Big]. \tag{31}$$

It is readily derived that $l^{k,*} \geq N$ if and only if

$$\theta \leq \underline{\theta}^k = \frac{2MNA}{(M+1)(N+1)(4N\lambda_k - N - \lambda_k)}. \tag{32}$$

Similarly, $l^{k,*} \geq L^k$ if and only if

$$\theta \geq \bar{\theta}^k = \frac{2MNA}{(M+1)(N+1)(N+\lambda_k)}. \tag{33}$$

As a corollary, under the optimal fixed fee contract the innovator licenses l^k firms with

$$l^k = \begin{cases} N & \theta \leq \underline{\theta}^k, \\ \frac{1}{4\lambda_k}\big[\frac{2MNA}{(M+1)(N+1)} + N + \lambda_k\big] & \theta \in (\underline{\theta}^k, \bar{\theta}^k), \\ L^k & \theta \in [\bar{\theta}^k, \bar{\bar{\theta}}^k). \end{cases} \tag{34}$$

7.4 The optimal licensing contract for a general k

In this section, we derive the conditions under which the innovator realizes strictly larger licensing revenues under a per-unit royalty than a fixed fee contract.

Before doing so, we have a closer look at the innovator's licensing income under both policies. From Lemma 3 and 5 it follows that the latter are respectively given by

$$\pi^{P,r,k} = \frac{MNA\theta}{(M+1)(N+1)} \tag{35}$$

[19] The first constraint, i.e., $l^k \leq N$, stems from the fact that the number of licensing contracts cannot exceed the number of firms in the industry. The second constraint, i.e., $l^k \leq L^k$, is due to the assumption that $\pi_i^{n,nl,k}(l^k) \geq 0$ $\forall l^k$ so that $f^k(l^k)$ is given by (7). Finally, we require $L^k > 1$ for the innovation to be non-drastic. Note that we restrict our attention to $l^k \leq L^k$ (or $\pi^{n,nl,k}(l^k) \geq 0$). This choice is primarily motivated by simplicity and comparability of our results to the standard framework (in the standard framework it is never optimal for the innovator to license $l^k > L^k$ firms; see, e.g., Kamien and Tauman (1984a)). Moreover, considering $l^k > L^k$ does not add any interesting additional results or insights to our analysis.

and

$$\pi_l^{P,f,k} = \begin{cases} N\theta^2\big(1-\frac{\lambda_k}{N}\big)\big[\frac{2MA}{(M+1)(N+1)\theta} - \lambda_k(2-\frac{1}{N})+1\big] & \theta \in (0,\underline{\theta}^k], \\ \frac{N\theta^2}{2\lambda_k}\big(1-\frac{\lambda_k}{N}\big)\big[\frac{MA}{(M+1)(N+1)\theta}+\frac{1}{2}+\frac{\lambda_k}{2N}\big]^2 & \theta \in (\underline{\theta}^k,\bar{\theta}^k), \\ \frac{MNA\theta}{(M+1)(N+1)\lambda_k}\big(1-\frac{\lambda_k}{N}\big)\big(1+\frac{\lambda_k}{N}\big) & \theta \in [\bar{\theta}^k,\bar{\bar{\theta}}^k). \end{cases} \tag{36}$$

Lemma 11 *Whereas the innovator's royalty licensing revenues are independent of the scarcity of the input factor, their fixed fee licensing income strictly decreases in the latter. That is,* $\partial\pi^{P,r,k}/\partial k = 0$ *and* $\partial\pi_l^{P,f,k}/\partial k < 0$.

Proof The first part of Lemma 11, $\frac{\partial\pi^{P,r,k}}{\partial k} = 0$, directly follows from (35). For the second part, $\frac{\partial\pi_l^{P,f,k}}{\partial k} < 0$, note first that $\frac{\partial\lambda_k}{\partial k} > 0$. It thus suffices to show that $\frac{\partial\pi_l^{P,f,k}}{\partial\lambda_k} < 0$.

It is immediate that $\frac{\partial\pi_l^{P,f,k}}{\partial\lambda_k} < 0$ for $\theta \in (0,\underline{\theta}^k]$ and $\theta \in [\bar{\theta}^k,\bar{\bar{\theta}}^k)$. Regarding $\theta \in (\underline{\theta}^k,\bar{\theta}^k)$, note that

$$\frac{\partial\pi_{l^*}^{P,f,k}}{\partial\lambda_k} = \frac{\theta^2}{2}\Big\{\Big(-\frac{N}{\lambda_k^2}\Big)\Big[\frac{MA}{(M+1)(N+1)\theta}+\frac{1}{2}+\frac{\lambda_k}{2N}\Big]^2 + \frac{1}{N}\Big(\frac{N}{\lambda_k}-1\Big)\Big[\frac{MA}{(M+1)(N+1)\theta}+\frac{1}{2}+\frac{\lambda_k}{2N}\Big]\Big\}. \tag{37}$$

Consequently, $\frac{\partial\pi_{l^*}^{P,f,k}}{\partial\lambda_k} < 0$ if and only if

$$\frac{N}{\lambda_k} < 1 + \Big(\frac{N}{\lambda_k}\Big)^2\Big[\frac{MA}{(M+1)(N+1)\theta}+\frac{1}{2}+\frac{\lambda_k}{2N}\Big]. \tag{38}$$

Taking into account that $N > 1$, $\lambda_k \leq 1$ and $\theta < \frac{MNA}{(M+1)(N+1)\lambda_k}$ it is readily verified that

$$\frac{N}{\lambda_k} < \Big(\frac{N}{\lambda_k}\Big)^2\Big[\frac{MA}{(M+1)(N+1)\theta}+\frac{1}{2}+\frac{\lambda_k}{2N}\Big]. \tag{39}$$

and therefore that $\frac{\partial\pi_{l^*}^{P,f,k}}{\partial\lambda_k} < 0$. □

In the remainder of this section, we compare the innovator's licensing revenues across policies for minor, intermediate and large innovations.

7.4.1 Minor innovations

Lemma 12 *Minor innovations (i.e.,* $\theta \in (0,\underline{\theta}^k]$ *and* $l^k = N$*) are optimally licensed via per-unit royalty contracts if and only if* $\theta \in [\theta_1^k,\underline{\theta}^k]$ *(with* $\theta_1^k = \frac{MNA(N-2\lambda_k)}{(M+1)(N+1)(N-\lambda_k)(2N\lambda_k-N-\lambda_k)}$*) and* $N \leq \lambda_k(4\lambda_k-1)$*. As a corollary, there are no integer values for* M *and* N *such that* $\pi^{P,r,0} \geq \pi_N^{P,f,0}$*. Moreover,* $\pi^{P,r,\infty} \geq \pi_N^{P,f,\infty}$ *if and only if* $N \leq 3$ *and* $\theta \in [\theta_1^\infty,\underline{\theta}^\infty]$.

Proof For $\theta \in (0,\underline{\theta}^k]$, $\pi^{P,r,k} \geq \pi_N^{P,f,k}$ if and only if

$$\theta \geq \theta_1^k = \frac{MNA(N-2\lambda_k)}{(M+1)(N+1)(N-\lambda_k)(2N\lambda_k-N-\lambda_k)}. \tag{40}$$

Note that $2N\lambda_k - N - \lambda_k > 0 \; \forall \lambda_k$ and $M, N > 1$.

For there to exist a value of θ such that $\theta \in [\theta_1, \underline{\theta}^k]$, $\theta_1 \leq \underline{\theta}^k$ or

$$N \leq \lambda_k(4\lambda_k - 1) \tag{41}$$

has to hold. For $k \to \infty$, $\lambda_k \to 1$ so that the condition reduces to $N \leq 3$. □

7.4.2 Intermediate innovations

Lemma 13 *Intermediate innovations (i.e., $\theta \in (\underline{\theta}^k, \bar{\theta}^k)$ and $l^k = l^{*,k}$) are optimally licensed via per-unit royalty contracts if and only if $\theta \in (\max\{\theta^k_{2,-}, \underline{\theta}^k\}, \bar{\theta}^k)$ (with $\theta^k_{2,-} = \bar{\theta}^k * \big[\frac{\lambda_k^2 - N^2 + 2N^2\lambda_k - 2N\sqrt{N^2\lambda_k(\lambda_k-1)+\lambda_k^3}}{N^2-\lambda_k^2}\big]$) and $\lambda_k \geq N(\sqrt{N^2+2} - N)/2$. As a corollary, $\pi^{P,r,0} \geq \pi^{P,f,0}_{l^*}$ if and only if $M = 1, N = 2$ and $\theta \in (\underline{\theta}^0, \bar{\theta}^0)$ (integer solution). Moreover, $\pi^{P,r,\infty} \geq \pi^{P,f,\infty}_{l^*}$ if and only if either $N \leq 3$ and $\theta \in (\underline{\theta}^\infty, \bar{\theta}^\infty)$ or $N > 3$ and $\theta \in (\theta^\infty_{2,-}, \bar{\theta}^\infty)$.*

Proof

For $\theta \in (\underline{\theta}^k, \bar{\theta}^k)$, $\pi^{P,r,k} \geq \pi^{P,f,k}_{l^*}$ if and only if

$$\theta^2\Big(\frac{N+\lambda}{2N}\Big)^2 + \theta\Big[\frac{MA}{(M+1)(N+1)}\Big]\Big[\frac{1}{N(N-\lambda)}\Big](N^2 - \lambda^2 - 2N^2\lambda) + \Big[\frac{MA}{(M+1)(N+1)}\Big]^2 \leq 0 \tag{42}$$

or

$$\theta \in [\theta^k_{2,-}, \theta^k_{2,+}] \tag{43}$$

with $\theta^k_{2,\pm} = \bar{\theta}^k * \big[\frac{\lambda_k^2 - N^2 + 2N^2\lambda_k \pm 2N\sqrt{N^2\lambda_k(\lambda_k-1)+\lambda_k^3}}{N^2-\lambda_k^2}\big]$. For the following analysis we impose $\lambda_k \geq N(\sqrt{N^2+4} - N)/2$ (positive discriminant). As a corollary, $\theta^k_{2,+} \geq 0$.

It remains to investigate the relationship between $\theta^k_{2,-}, \theta^k_{2,+}$, $\underline{\theta}^k$ and $\bar{\theta}^k$ in order to derive the relevant bounds for θ (i.e., the conditions on θ under which i) the innovation is licensed to $l^k = l^{*,k}$ firms under a fixed fee contract and ii) the innovator is strictly better off under a per-unit royalty than a fixed fee contract).

First, we observe that the relevant upper bound on θ is $\bar{\theta}^k$. To see this note that $\bar{\theta}^k \leq \theta^k_{2,+}$ reduces to

$$\frac{\lambda_k^4}{N^2} + \lambda_k^3 - 2\lambda_k^2 - N^2\lambda_k + N^2 \leq 0 \tag{44}$$

which is equivalent to $\lambda_k \geq N(\sqrt{4+N^2} - N)/2$.

Second, depending on the value of λ_k the relevant lower bound on θ is $\max\{\theta^k_{2,-}, \underline{\theta}^k\}$.[20] In particular, noticing that $\underline{\theta}^k = \bar{\theta}^k * \big(\frac{N+\lambda_k}{4N\lambda_k - N - \lambda_k}\big)$ one can express $\theta^k_{2,-} \leq \underline{\theta}^k$ as

$$\bar{\theta}^k * \Big[\frac{\lambda_k^2 - N^2 + 2N^2\lambda_k - 2N\sqrt{N^2\lambda_k(\lambda_k-1)+\lambda_k^3}}{N^2-\lambda_k^2}\Big] \leq \bar{\theta}^k * \Big[\frac{N+\lambda_k}{4N\lambda_k - N - \lambda_k}\Big]. \tag{45}$$

[20]Note that $\bar{\theta}^k \geq \underline{\theta}^k$ and $\bar{\theta}^k \geq \theta^k_{2,-}$ for any $\lambda_k \geq N(\sqrt{4+N^2} - N)/2$.

Rearrangement yields

$$(4N\lambda_k - N - \lambda_k)\big[N\lambda_k - \sqrt{N^2\lambda_k(\lambda_k - 1) + \lambda_k^3}\big] \leq 2\lambda_k(N^2 - \lambda_k^2). \tag{46}$$

It is easily seen that for $k \to \infty$ the inequality reduces to $N \leq 3$. □

7.4.3 Large innovations

Lemma 14 *Large innovations (i.e., $\theta \in [\bar{\theta}^k, \bar{\bar{\theta}}^k)$ and $l^k = L^k$) are optimally licensed via per-unit royalty licensing contracts if and only if $\theta \in [\bar{\theta}^k, \bar{\bar{\theta}}^k)$ and $(\frac{\lambda_k}{N})^2 + \lambda_k \geq 1$. As a corollary, $\pi^{P,r,0} \geq \pi_L^{P,f,0}$ if and only if $M = 1, N = 2$ and $\theta \in [\bar{\theta}^0, \bar{\bar{\theta}}^0)$ (integer solution). Moreover, $\pi^{P,r,\infty} \geq \pi_L^{P,f,\infty}$ if and only if $\theta \in [\bar{\theta}^\infty, \bar{\bar{\theta}}^\infty)$.*

Proof For $\theta \in [\bar{\theta}^k, \bar{\bar{\theta}}^k)$, $\pi^{P,r,k} \geq \pi_L^{P,f,k}$ if and only if

$$1 \geq \frac{1 - \big(\frac{\lambda_k}{N}\big)^2}{\lambda_k}. \tag{47}$$

Clearly, for $k \to \infty$ (47) is always satisfied. □

7.5 Welfare outcomes

In a first step, we summarize consumer and producer surplus for a per-unit royalty and a fixed fee licensing contract (see Table 2). We then compute aggregate welfare for either type of licensing policy by summing consumer surplus, producer surplus and licensing revenues. In a second step, we compare aggregate welfare across licensing policies under strict ($k \to \infty$) and soft capacity constraints ($k \in (0, \infty)$).

		Per-unit royalty	**Fixed fee**
Consumer surplus	CS^k	$\frac{1}{2}(X_{pre})^2$	$\frac{1}{2}(X^k)^2$
Producer surplus	$\Pi^{m,k}$	$M\pi_i^m = w_{pre}X_{pre}$	$M\pi_i^{m,k} = M\big[w^k x_i^k - \frac{k}{2}(x_i^k - x_{pre})^2\big]$
	$\Pi^{n,k}$	$N\pi_i^n = \frac{(X_{pre})^2}{N}$	$l^k\pi_i^{n,nl,k}(l^k - 1) + (N - l^k)\pi_i^{n,nl,k}(l^k)$

Table 2: Consumer and producer surplus for per-unit royalty and fixed fee contracts.

Based on the results in Table 2 we derive

$$\begin{aligned} W_{royalty}^k &= \frac{1}{2}(X_{pre})^2 + w_{pre}X_{pre} + \frac{(X_{pre})^2}{N} + X_{pre}\theta, \\ W_{fee}^k &= \frac{1}{2}(X^k)^2 + w^k X^k - \frac{Mk}{2}(x_i^k - x_{pre})^2 + (N - l^k)\Big(\frac{X^k}{N} - \theta\frac{l^k}{N}\lambda_k\Big)^2 + l^k\Big(\frac{X^k}{N} + \theta\big(1 - \frac{l^k}{N}\lambda_k\big)\Big)^2. \end{aligned} \tag{48}$$

Lemma 15 *Under strict capacity constraints, per-unit royalty contracts weakly dominate fixed fee contracts in terms of aggregate welfare. In particular, $W_{royalty}^\infty = W_{fee}^\infty$ for either $\theta \in (0, \underline{\theta}^\infty]$ or $\theta \in (\bar{\theta}^\infty, \bar{\bar{\theta}}^\infty]$, whereas $W_{royalty}^\infty > W_{fee}^\infty$ for $\theta \in (\underline{\theta}^\infty, \bar{\theta}^\infty)$.*

Proof Note that for $k \to \infty$, $\lambda_k \to 1$ and $X^k \to X_{pre}$. As a consequence, $W^{\infty}_{royalty} \geq W^{\infty}_{fee}$ reduces to

$$\frac{X_{pre}}{\theta}(N - l^{\infty}) \geq l^{\infty}(N - l^{\infty}). \tag{49}$$

It is immediate that for $\theta \in (0, \underline{\theta}^{\infty}]$ (i.e., for $l^{\infty} = N$) $W_{royalty} = W_{fee}$. For $\theta > \underline{\theta}^{\infty}$, the constraint simplifies to

$$\theta \leq \frac{X_{pre}}{l^{\infty}}. \tag{50}$$

Note first that $\theta < \frac{X_{pre}}{l^{*,\infty}}$ so that $W^{\infty}_{royalty} > W^{\infty}_{fee}$ for $\theta \in (\underline{\theta}^{\infty}, \bar{\theta}^{\infty})$ (i.e., for $l^{\infty} = l^{*,\infty}$). Second, $\theta = \frac{X_{pre}}{L^{\infty}}$ which implies that $W^{\infty}_{royalty} = W^{\infty}_{fee}$ for $\theta \in [\bar{\theta}^{\infty}, \bar{\bar{\theta}}^{\infty})$ (i.e., for $l^{\infty} = L^{\infty}$). □

Lemma 16 *Under soft capacity constraints, either per-unit royalty or fixed fee contracts maximize aggregate welfare. In particular,* $W^k_{fee} > W^k_{royalty}$ *for either* $\theta \in (0, \underline{\theta}^k]$ *or* $\theta \in (\bar{\theta}^k, \bar{\bar{\theta}}^k]$, *whereas* $W^k_{royalty} \geq W^k_{fee}$ *for* $\theta \in (\underline{\theta}^k, \bar{\theta}^k)$.

Proof First, it is clear that $W^k_{royalty}$ is independent of k and by this unaffected by a change in the strength of capacity constraints, i.e., $\frac{\partial W^k_{royalty}}{\partial k} = 0$. Second, W^k_{fee} increases as capacity constraints soften, i.e., $\frac{\partial W^k_{fee}}{\partial k} < 0$. With Lemma 15 in mind, Lemma 16 immediately follows. □

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